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HEAT TRANSFER ANALYSIS FOR UNSTEADY HIGH VELOCITY PIPE FLOW

PART I. ON MINIMIZATION OF TEMPERATURE DISTORTION IN THE THERMOCOUPLE CAVITY. PART II. IMPROVED ACCURACY IN THE PREDICTION OF SURFACE HEAT FLUX AND TEMPERATURE BY INTRINSIC THERMOCOUPLE. PART III. PREDICTION OF TRANSIENT SURFACE HEAT FLUX AND TEMPERATURE ON A HOLLOW CYLINDER

IOWA INSTITUTE OF HYDRAULIC RESEARCH
IOWA CITY, IOWA

OCTOBER 1976

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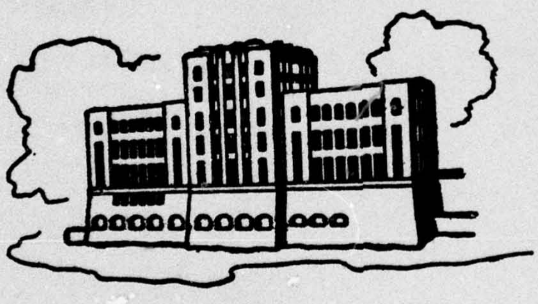
Part III Prediction of Transient Surface Heat Flux and Temperature on a Hollow Cylinder

by

Ching Jen Chen, Jenq Shing Chiou, Peter Li
and Hsai Yin Lee

Sponsored by

U.S. Army Research Office
Grant Number DAA-G29-76-G-0123



IIHR Report No. 195

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Iowa City, Iowa

October 1976

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ABSTRACT

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ACKNOWLEDGEMENTS

The investigators would like to thank Mr. D.M. Thomsen of General Rodman Laboratory, Rock Island Arsenal, Rock Island, Illinois for his constant participation in the course of the research.

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PART I ON MINIMIZATION OF TEMPERATURE DISTORTION IN THE THERMOCOUPLE CAVITY

I. INTRODUCTION

A direct measurement of transient surface temperature and heat flux is often difficult. For example, a surface involves two modes of heat transfer, say, radiative and convective heat transfer. In this case if the measuring probe has a different radiative property than that of the surface, erroneous measurements will result. A piston or projectile sliding over the cylindrical surface is another case where the direct measurement at the surface is difficult. A surface involving melting or ablation is also difficult to make direct measurement. Therefore, indirect estimation by inverting the temperature history inside the heat conducting solid as measured by a thermocouple is often used for prediction of the surface temperature and heat flux. Beck [1], Hernning and Parker [2], Frank [3], Imber and Khan [4], Stolz [5], Chen and Thomsen [6] have developed different inversion solutions for this purpose. All of these solutions assumed that the cavity drilled into the solid does not distort the true temperature distribution. Thus, it is important that the temperature measurement by an interior probe is accurate and involves least distortion or error. Theoretically Beck [7], Masters and Stein [8], Burnett [9], and Chen and Li [10] studied the distortion of the temperature field in the present of thermocouple and its cavity. Experimentally Chen and Danh [11] showed that appreciable distortion, say 10%, of temperature field may exist for a normal implant of the thermocouple into a solid body. From studies of Chen and Li [10] and Beck [7] they found that with a proper combination of the thermocouple cavity diameter, its depth, and the thermocouple material and its diameter

the distortion of temperature field with respect to space or time can be minimized if not eliminated. In this report we study the optimum combination of these parameters such that at a given situation one knows what is the best combination to use and what is the magnitude of the temperature distortion.

II. FORMULATION OF PROBLEM

In the present study we consider a disk depicted in Figure 1 which has a thickness D and is drilled a cavity of a diameter to a depth of ε distance from the heating surface. The heat flux Q is assumed to be constant and the upper surface of the disk is assumed to be insulated. A thermocouple of a diameter d_t is then welded on the cavity base. Furthermore, the disk may be thought to approximate a cut out from a hollow cylinder if the radius of the disk is small compared with the radius of the cylinder. The diameter of the disk is choosen to be 2D outside which the temperature distortion due to the thermocouple cavity becomes negligiable. For this to be true one needs to restrict the ratio of cavity diameter to the disk diameter d/2D be small. The unfilled cavity can be air or insulating material.

The basic idea to minimize or to eliminate the temperature distortion is based on a proper choice of the thermocouple size and the material which has a higher thermal conductivity than that of the disk so as to conduct more heat away at the cavity base balancing the insulation effect of the insulator in the cavity.

Let X and Y be respectively the coordinate along and normal to the heated disk surface and the Y axis coincide with the axis of the cavity.

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Let X and Y be respectively the coordinate along and normal to the heated disk surface and the Y axis coincide with the axis of the cavity.

The thermal conductivity is assumed to be constant and the temperature distribution is axisymmetrical. The governing equations for the transient heat conduction in dimensionless form are:

for the disk (subscript "1", see Figure 2)

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial x^2} + \frac{1}{x}\frac{\partial\theta}{\partial x} + \frac{\partial^2\theta}{\partial y^2} \tag{1}$$

for the insulating material in the cavity (subscript "2")

$$\frac{\partial\theta}{\partial\tau} = \frac{\alpha_2}{\alpha_1}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{1}{x}\frac{\partial\theta}{\partial x} + \frac{\partial^2\theta}{\partial y^2}\right) \tag{2}$$

for the thermocouple (subscript "3")

$$\frac{\partial\theta}{\partial\tau} = \frac{\alpha_3}{\alpha_1}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{1}{x}\frac{\partial\theta}{\partial x} + \frac{\partial^2\theta}{\partial y^2}\right) \tag{3}$$

where $\tau = \alpha_1 t/D^2$ is the dimensionless time, $x = X/D$ the dimensionless radial coordinate, $y = Y/D$, the dimensionless distance normal to the heated surface. α_1, α_2, and α_3 are respectively the thermal diffusivity of the disk, insulating material, and the thermocouple. The dimensionless temperature θ is defined as $T\kappa_1/QD$ where T is the temperature above the initial, uniform temperature.

The intial temperature of the disk is then

$$\theta\ (x,\ y,\ 0) = 0 \tag{4}$$

The boundary conditions, see Figure 2, are:

a constant heat flux at the lower surface,

$$y = 0 \qquad \left.\frac{\partial\theta}{\partial y}\right|_{y=0} = 1 \tag{5}$$

the insulation at the upper surface,

$$y = 0 \qquad \left.\frac{\partial\theta}{\partial y}\right|_{y=0} = 0 \tag{6}$$

the zero temperature distortion at the edge of the disk,

$$x = 1 \qquad \left.\frac{\partial\theta}{\partial x}\right|_{x=1} = 0 \tag{7}$$

and the axisymmetric condition at the cavity axis

$$x = 0 \qquad \left.\frac{\partial\theta}{\partial x}\right|_{x=0} = 0 \tag{8}$$

The condition (7) of the zero temperature distortion was verified by Chen and Li [10] in their early calculation when the cavity diameter is one tenth of the disk diameter. In addition to the above boundary condition the temperature and heat flux at the interface of the disk, thermocouple, and insulating material are taken to be continuous.

III ANALYSIS

There are five parameters that can be varied for the present analysis. They are (a) the dimensionless distance from the base of the cavity to the heated surface ε/D, (b) the size of the cavity d/D, (c) the ratio of the thermocouple diameter to that of the cavity, d_t/d.

(d) the thermal conductivity ratio κ_2/κ_1, and κ_3/κ_1 which comes from the continuity of heat flux at interfaces. (e) the ratio of the product of density and specific heat $\rho_3 c_3/\rho_1 c_1$ (or equivalently to the ratio of of thermal diffisivity α_3/α_2 or α_2/α_1). The subscripts 1, 2 and 3 denote the disk, insulation and thermocouple materials.

Because of the complexity of the geometry and the multicity of material the method of finite element technique as discussed by Wilson and Nikel [12] is adapted with the aid of a computer program developed by Wilson [13]. The present problem is subdivided into finite element as required by the method, see Figure 2. The dimensionless pie section is subdivided into 121 finite elements with 12 dividing lines on both coordinates. Each element is defined by four nodal points where nodal points are denoted by intersections of the dividing lines and numbered as shown in Figure 2. The material property corresponding to each element is then assigned to the program developed by Wilson [13] The solution at each nodal with respect to time is then obtained.

For numerical calculation three typical values of the distance from heated surface to the base of the cavity ε/D are chosen to be 0.04, 0.1 and 0.2. The cavity diameter is fixed at one tenth of the disk diameter. The thermocouple to cavity diameter ratio d_t/d is made to vary 0, 0.2, 0.4, 0.6, 0.8 and 1.0. Regarding the range of the ratio of the thermal conductivity and the ratio of the product of density and specific heat we surveyed these ratios for the commonly used thermocouples in Table 1 [14] and plotted in Figure 3. Figure 3b shows the ratio of

thermocouple conductivity to that of the disk κ_3/κ_1 versus the ratio of density - specific heat product ρ_3c_3/ρ_1c_1 where the value of the conductivity is taken to be the average value between 200 and 800°K. One sees that for most practical situations the ratio of density - specific heat product ρ_3c_3/ρ_1c_1 is approximately constant at 0.7 except when the conductivity ratio κ_3/κ_1 is small. Thus the value of ρ_3c_3/ρ_1c_1 and κ_3/κ_1 for calculation are chosen, as shown in triangular symbols of Figure 3b, to cover the practical range. The corresponding value for ρ_2c_2/ρ_1c_1 and κ_2/κ_1 for the insulation material are chosen to be fixed at 0.5 and 0.005 which is a typical value for Teflon insulating material and is also approximately the order of magnitude for air.

IV. RESULTS AND DISCUSSIONS

Numerical results of the calculations are presented in Tables 2 to 4 and Figures 4 to 9. The percentage error of temperature is defined as the distorted temperature divided by a reference temperature defined by QD/κ_1. Tables 2 to 4 give the percentage error of temperature distortion) as a function of time for different values of the parameters d_t/d (0 to 1.0), κ_3/κ_1 (0.5 to 10), ρ_3c_3/ρ_1c_1 (0.5 to 1.8) and ε/D (0.02 to 0.1).

Figures 4, 5 and 6 show the three typical temperature distributions in the steel disk near the thermocouple junction for the case $\varepsilon/D = 0.06$ $d/2D = 0.1$ at the time $\tau = 0.08$. Figure 4 is the temperature distribution when the cavity is filled entirely with the insulation material (Teflon "2" $\kappa_2/\kappa_1 = 0.005$, $\rho_2c_2/\rho_1c_1 = 0.5$). The dimensionless isotherms

$T\kappa_1/QD$ shows the distortion of temperature distribution. One sees that the insulation effect on the heat transfer near the cavity base not only creates a much higher junction temperature of $T\kappa_1/QD = 0.342$ than the undistorted one of 0.255 at the edge of the disk giving an error of 8.7% but also causes a hot spot at the heating surface with a higher temperature of $T\kappa_1/QD = 0.374$ over the undistorted one of 0.311. On the other hand the temperature distribution in the insulation material is much lower than the true temperature creating a large temperature gradient at the base of the cavity. Figure 5, contrary to Figure 4, is the temperature distribution when the cavity is completely filled with thermocouple material whose thermal conductivity is ten times larger than the disk material e.g., copper versus steel. Now over-conduction of heat by the thermocouple has created a cold spot at the base of the cavity giving, $T\kappa_1/QD$ of 0.17 versus the undistorted one of 0.255 with an error of 8.5% as well as at the heating surface with $T\kappa_1/QD$ of 0.236 versus 0.311. The temperature distribution in the thermocouple now becomes higher than the undistorted one in the disk. By properly choosing the ratio of thermocouple diameter to that of the cavity one may minimize these distortions of the temperature response at the base of the cavity. This is shown in Figure 6 where the thermocouple diameter d_t is choosen to be 0.4 of the cavity diameter d with $\kappa_3/\kappa_1 = 10$, e.g. copper-steel combination. Figure 6 shows that distortions at the base of the cavity and at the heating surface are almost eliminated giving the temperature $T\kappa_1/QD$ of 0.245 and 0.320 with respectively the error of 1% and 0.9%.

In order to examine the details of the distorted temperature response at the base of the cavity we tabulated the results in Tables 2, 3 and 4 and plotted the error percentage as function of the ratio of the thermocouple diameter to that of the cavity for different cavity depth, time, and thermal conductivity in Figures 7, 8 and 9. In these Figures the ρc ratio ranges from 0.7 to 1.3 which covers most of the practical application.

In the case when the ρc ratio is equal to or less than one the error in temperature response at the base of the cavity in these figures are all positive or overheat for $\kappa_3/\kappa_1 \leq 1$. This is because the thermal conductivity of the thermocouple is less than that of the disk and the heat capacity ρc of the thermocouple is also small. Therefore, no extra conduction of heat can be achieved by the thermocouple to compensate for the blocking of the heat transfer by the insulation material in the cavity. On the other hand, if $\kappa_3/\kappa_1 > 1$ the error of temperature varies from positive value for $d_t/d = 0$ to some negative value as d_t/d approaches 1. Thus for $\kappa_3/\kappa_1 > 1$ a properly chosen combination of thermocouple and insulation material can minimize the error. For example, in Figure 7 a combination of $\kappa_3/\kappa_1 = 10$, $\rho_3 c_3/\rho_1 c_1 = 0.75$, $\varepsilon/D = 0.1$ and $d_t/d = 0.5$ produces almost negligible error. This combination shown to be optimum at $\tau = 1.0$ in Figure 7 is also optimum for other time periods (see Table 2). Therefore once an optimum combination of parameters is chosen it is valid throughout the entire transient period of an experiment. From Figures 7, 8 and 9 one can also see that the optimum ratio of d_t/d which gives zero temperature error decreases as the κ_3/κ_1 ratio increases.

This implies that for the thermocouple with a larger thermal conductivity a smaller diameter is sufficient to eliminate the temperature distortion. The result shown in Figures 7, 8 and 9 can in general be adapted for use in practical application to choose the size of thermocouple and cavity, the thermocouple material and the depth of the cavity to be drilled.

As mentioned earlier that when $\kappa_3/\kappa_1 \leq 1$ and $\rho_3c_3/\rho_1c_1 \leq 1$ the error of the temperature response at the base of the cavity are all positive. However we found (see Tables 2.2, 3.2 and 4.2) that if ρ_3c_3/ρ_1c_1 ratio is made large enough during the transient period the error of temperature response at the base of the cavity may indeed become negative even when $\kappa_3/\kappa_1 \leq 1$. Physically although the thermal conductivity of the thermocouple κ_3 is smaller than that of the disk material, but with a larger heat capicitance $\rho_3c_3/\rho_1c_1 > 1$ the thermocouple is still capable of absorbing extra heat flux and hence eliminates the temperature distortion at the cavity base during the transient period. To illustrate this fact we examine Figure 7 (or see Table 4.2) for the data of $\kappa_3/\kappa_1 = 1$ and $\rho_3c_3/\rho_1c_1 = 1.3$. One sees that when $d_t/d = 1$ the temperature distortion can indeed be negative. Therefore, if d_t/d are choosen between 0.8 and 1 the error can be minimized. However one must keep in mind that the elimination of error by heat capacitance can work during the transient period only for once a steady state conduction is established the heat capacity ρc will no longer have any effect and over heating at the cavity base eventually will develop. This can be seen best from the governing equation (1) that at steady state the unsteady term which contains ρc product is zero and is not a parameter affecting the distortion.

Another important fact that should be mentioned is that in general the optimum choice of d_t/d ratio for given κ_3/κ_1 and ρc ratio does not vary very much with the variation of ε/D ratio. The insensitivity of the optimum d_t/d ratio to the ε/D ratio ranging from 0.02 to 0.1 means that the distortion of the temperature is insensitive to the cavity depth or the thickness of the disk. This fact was already pointed out by Chen and Danh [11] in their experiment that the temperature distortion at the base of the cavity is more sensitive to the variation of the cavity diameter than the depth of the cavity drilled.

As an example of a practical application, let us consider a measurement of the transient temperature response of an engine block made of aluminum. From Figure 3b we know that aluminum has high thermal conductivity. Therefore copper-constantan thermalcouple which has a higher thermal conductivity than aluminum should be chosen. For this material combination we have $\kappa_3/\kappa_1 = 1.69$ $\rho_3c_3/\rho_1c_1 = 1.3$. Now if the thermocouple cavity is drilled such that $\varepsilon/D = 0.1$ then from Figure 7 interpolating between $\kappa_3/\kappa_1 = 2$ and 1 for $\rho_3c_3/\rho_1c_1 = 1.3$ we find that the optimum d_t/d for $\kappa_3/\kappa_1 = 1.69$ is approximately 0.7

One disadvantage of invoking finite element analysis is that the result does not give a clear functional relation among the parameters involved. In an attempt to obtain a simple and useful relation to relate the various parameters we note the following fact and result: (a) the optimum d_t/d ratio for zero temperature distortion is a strong function of κ_3/κ_1 and ρc ratio but is relatively insensitive to the ε/D ratio, (b) from the theoretical reasoning the d_t/d ratio is independent

of ρc ratio if the problem is steady state. A simple steady one dimensional analysis in which the thermocouple and the insulation material in the cavity is made to conduct the same amount of heat that would be transferred without the cavity gives the relation

$$d_t/d = \sqrt{(\kappa_1 - \kappa_2)/(\kappa_3 - \kappa_2)} \tag{9}$$

Using the above equation as a base we find that for the transient heat conduction as calculated by the finite element method the following equation (10) correlates very well with the optimum d_t/d ratio.

$$d_t/d = (\rho_3 c_3/\rho_1 c_1)^{0.3} \sqrt{(\kappa_1 - \kappa_2)/(\kappa_3 - \kappa_2)} \tag{10}$$

Equation (10) gives an error or distortion of no more than two percentage points. In practice equation (10) may be used as a rule of thumb.

V. CONCLUSION

An analysis of the temperature distortion caused by the cavity drilled into a disk to accommodate the thermocouple has been studied. The calculation is carried out for the case of constant heat flux. It is shown that the temperature at the base of the cavity distorted from that without a cavity can be eliminated by a properly chosen combination of the ratio of the thermocouple diameter to the cavity diameter, d_t/d and the thermocouple material κ_3/κ_1. The optimum ratio of d_t/d can be found from Figures 7, 8 and 9 or Tables 2, 3 and 4, or approximately from equation (10). As a rule the thermocouple must be chosen

to have a higher thermal conductivity than that of the heat conducting solid. The cavity diameter should be as small as practically possible. For the case of time dependent surface heat flux the present result may be also used as a general guide.

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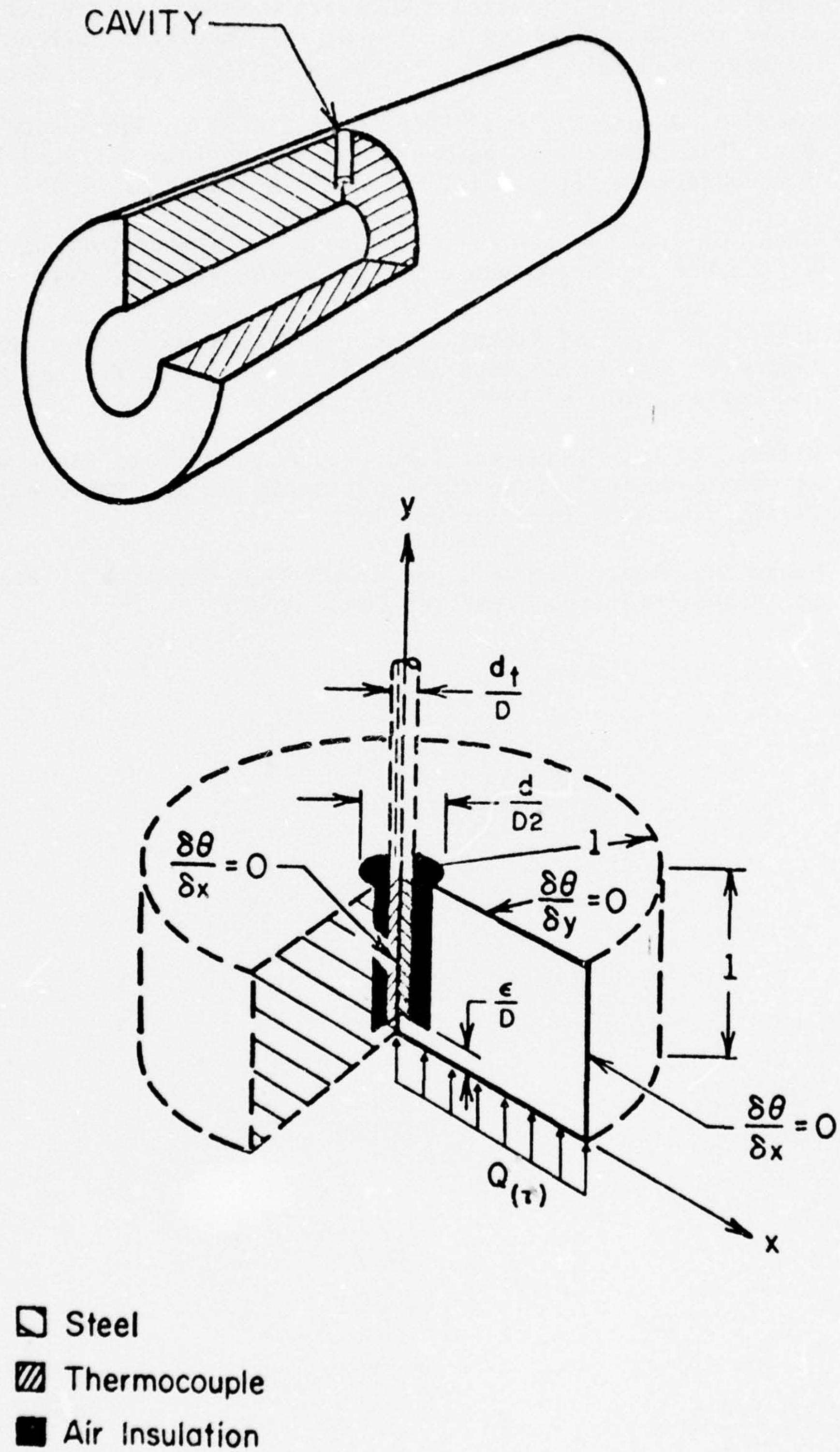

Figure 1 Geometric Representation of Problem

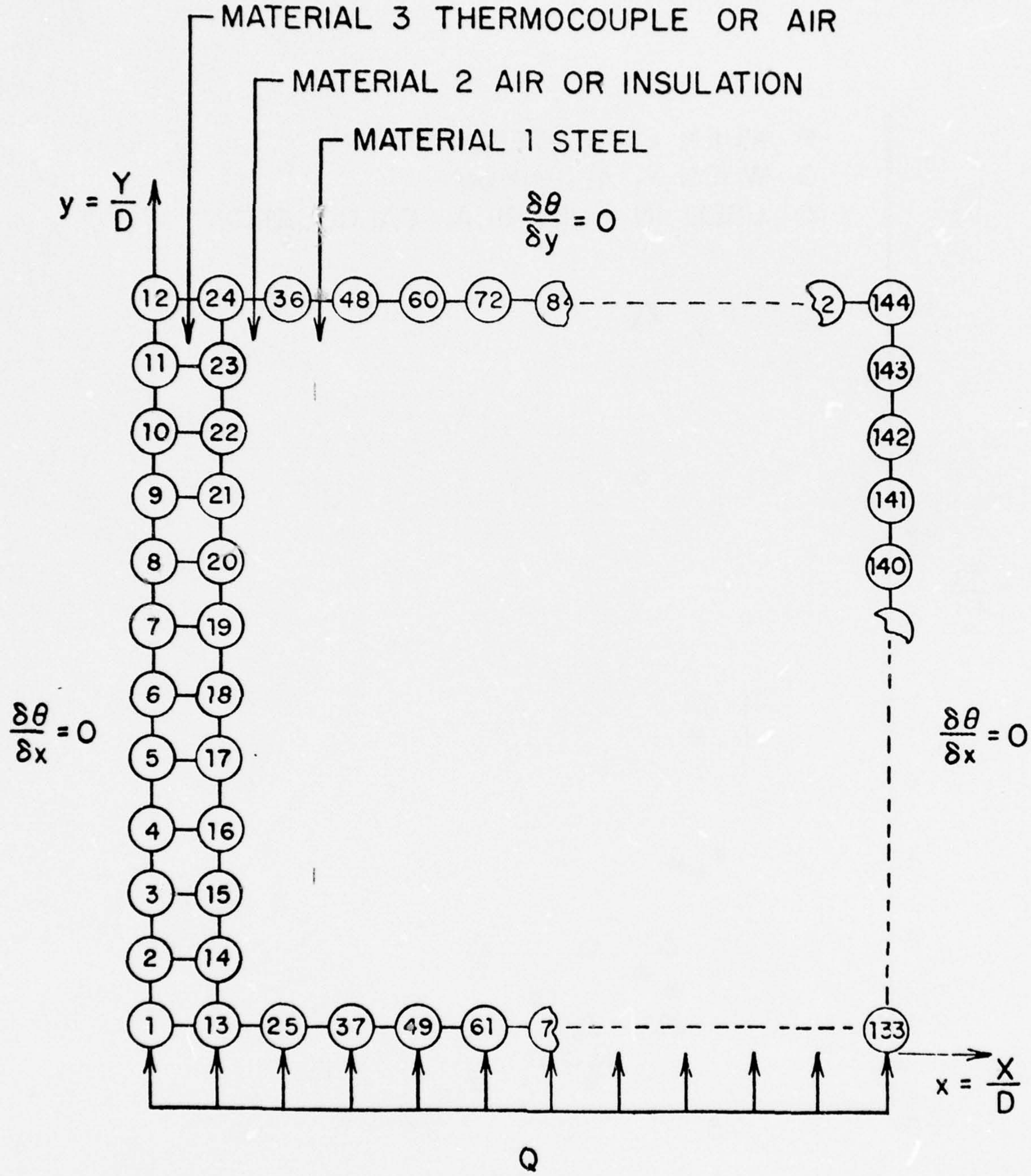

Figure 2 Finite Element Idealization

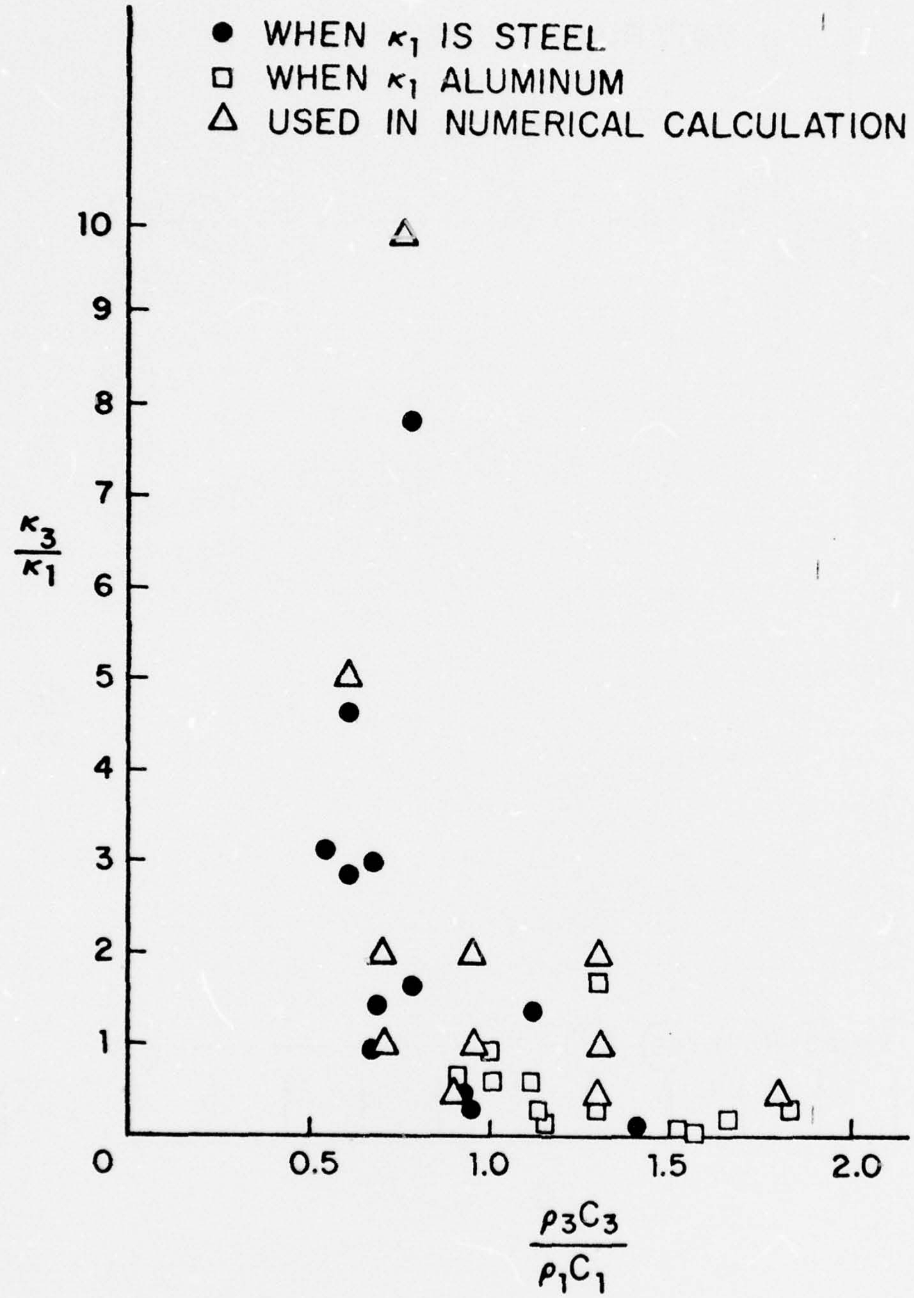

Figure 3b Variation of Thermal Conductivity Ratio and Density - Specific Heat Ratio

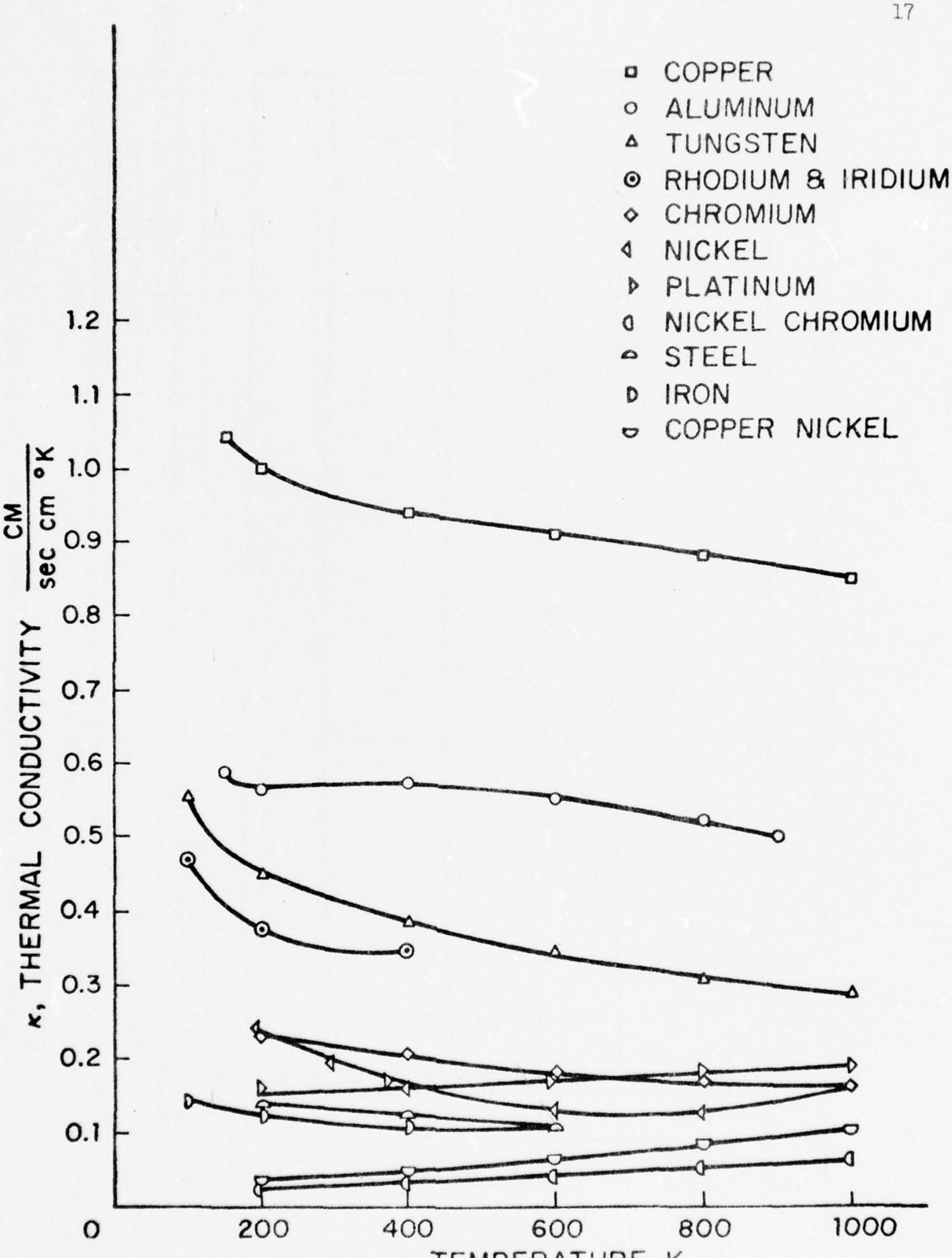

Figure 3a Thermal Conductivity of Thermocouple Materials

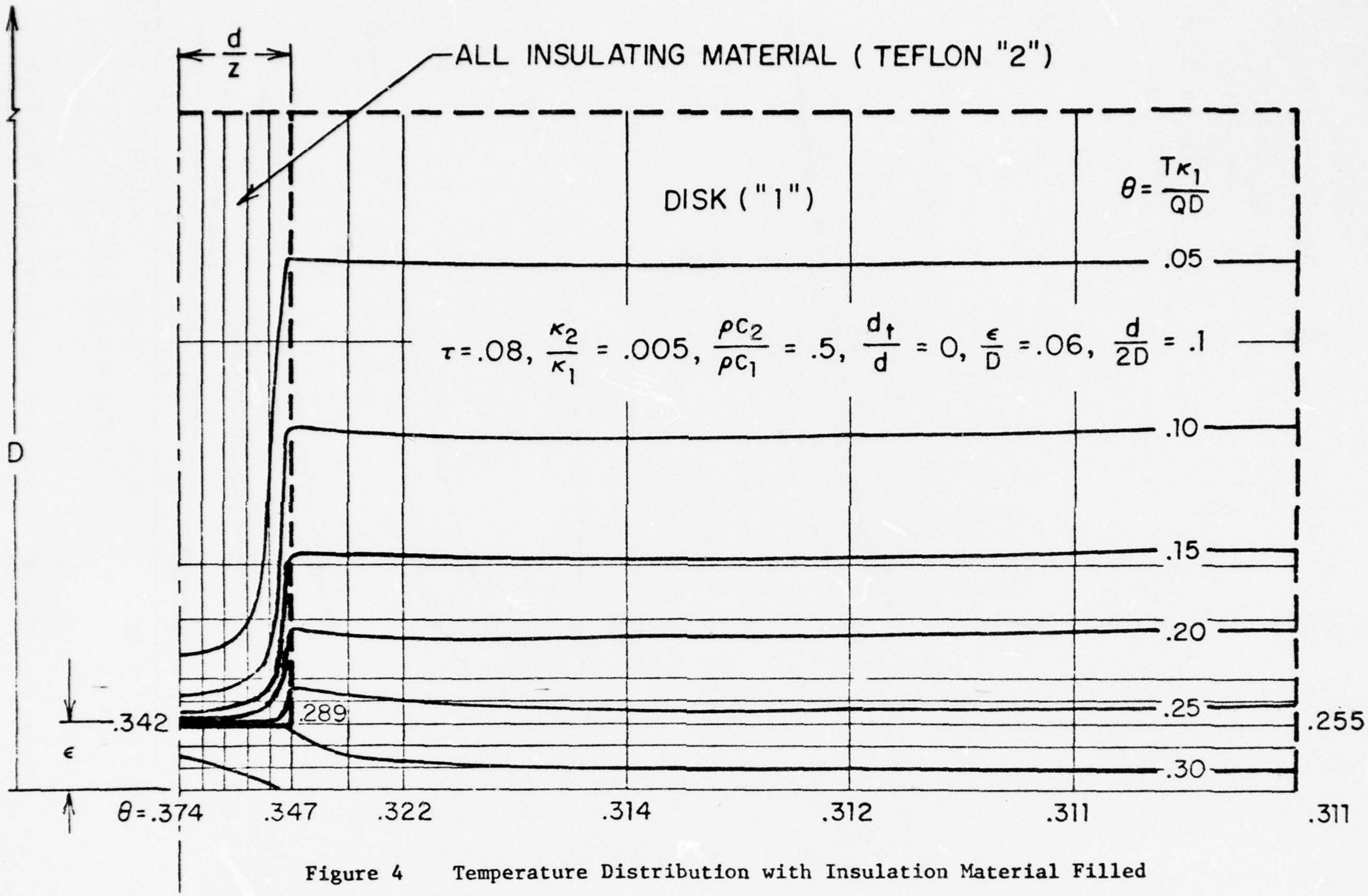

Figure 4 Temperature Distribution with Insulation Material Filled the Cavity

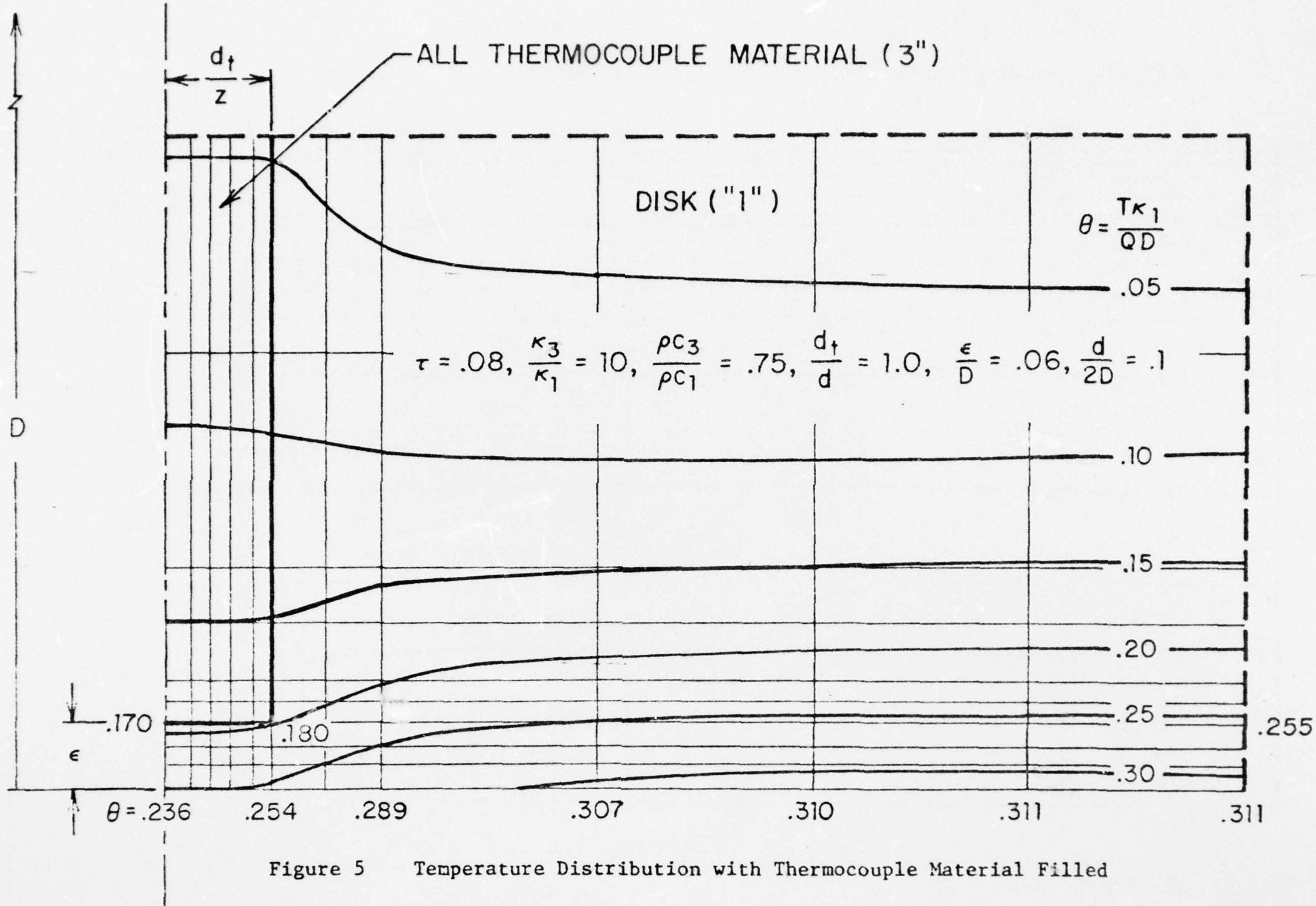

Figure 5 Temperature Distribution with Thermocouple Material Filled the Cavity

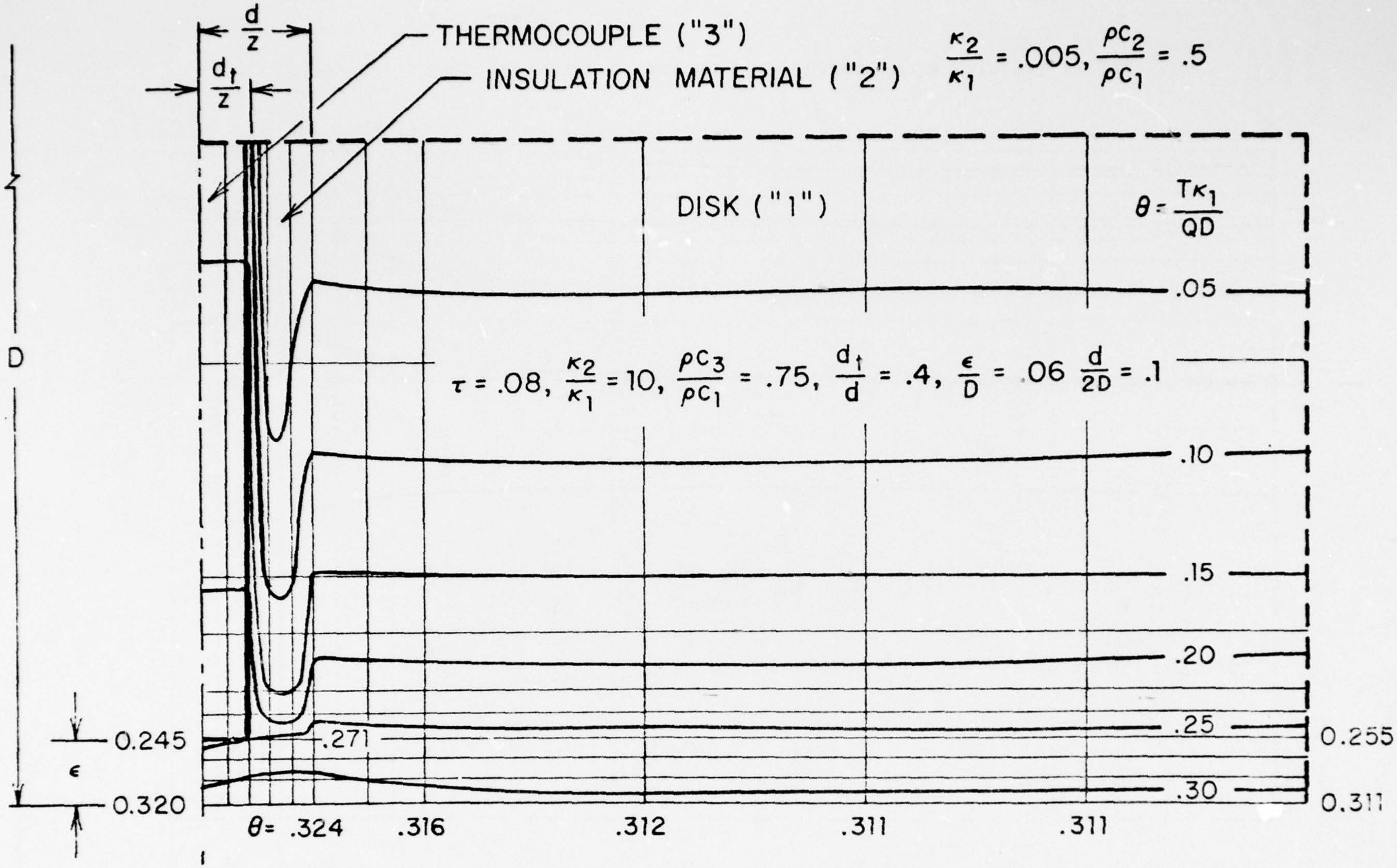

Figure 6 Temperature Distribution with Thermocouple Material Partially Filled the Cavity

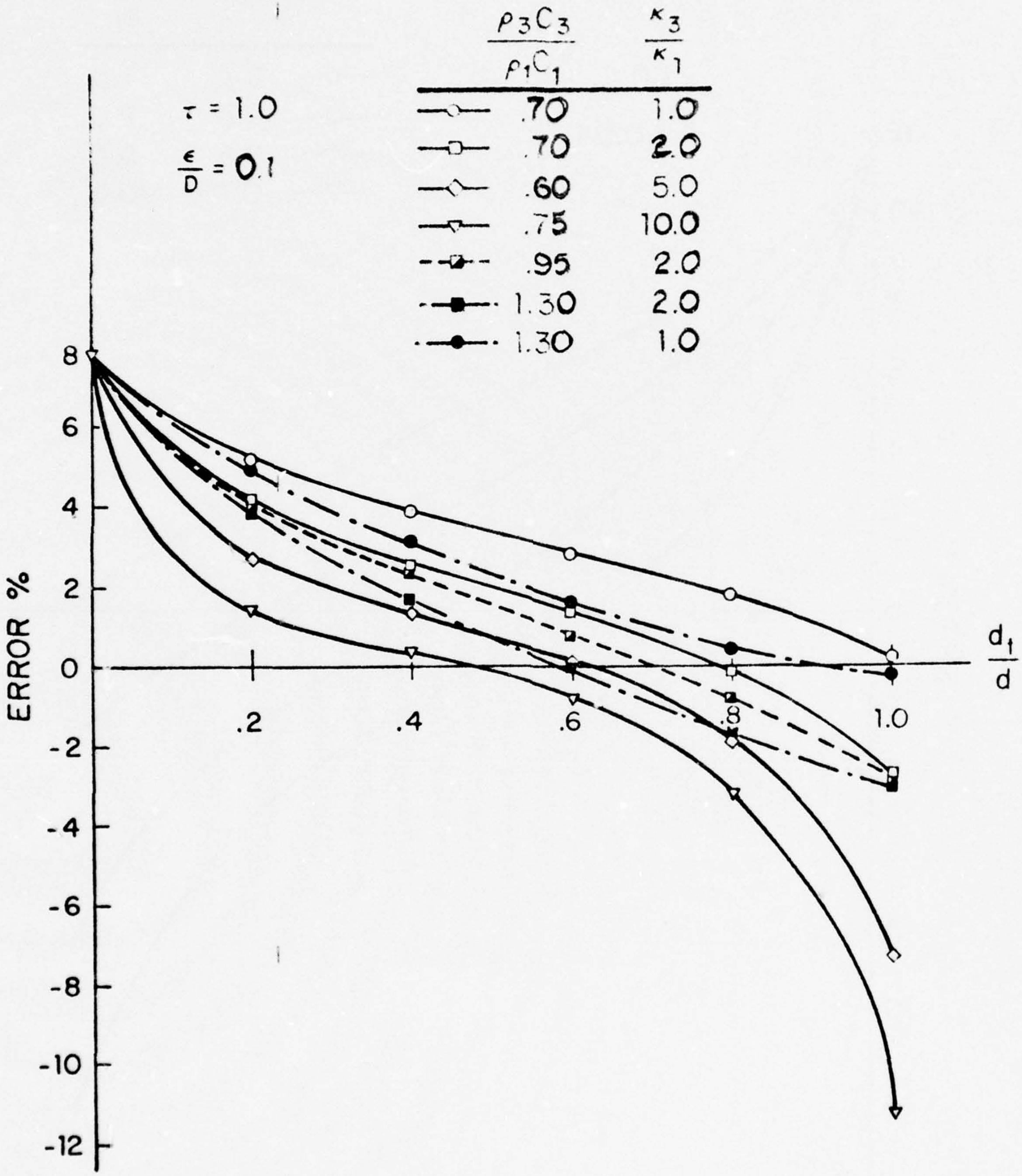

Figure 7 Percentage Error vs. d_t/d Ratio for Various κ_3/κ_1, and ρc Ratios $\epsilon/D = 0.1$

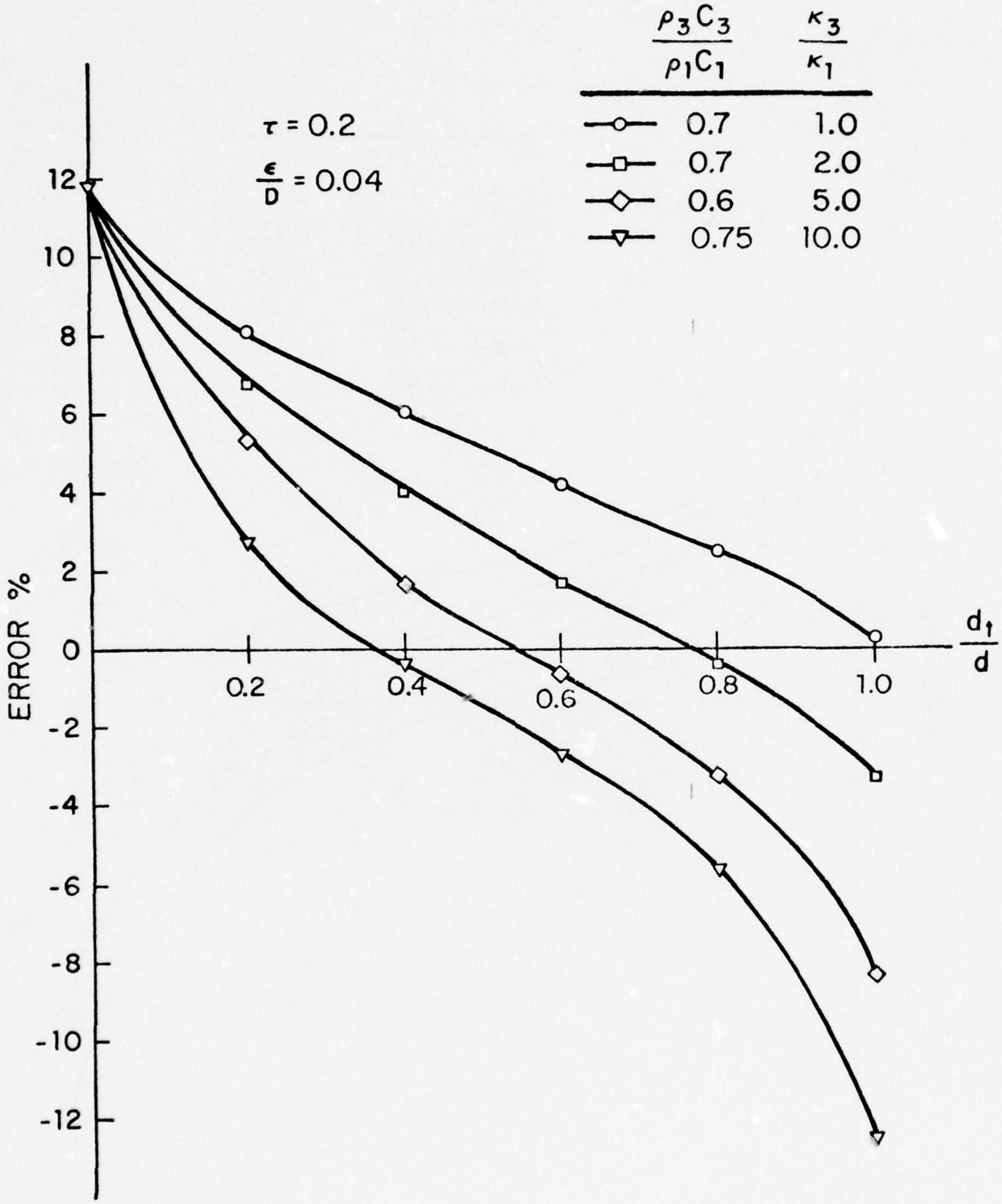

Figure 8 Percentage Error vs. d_t/d Ratio for Various κ_3/κ_1 and ρc Ratios $\epsilon/D = 0.04$

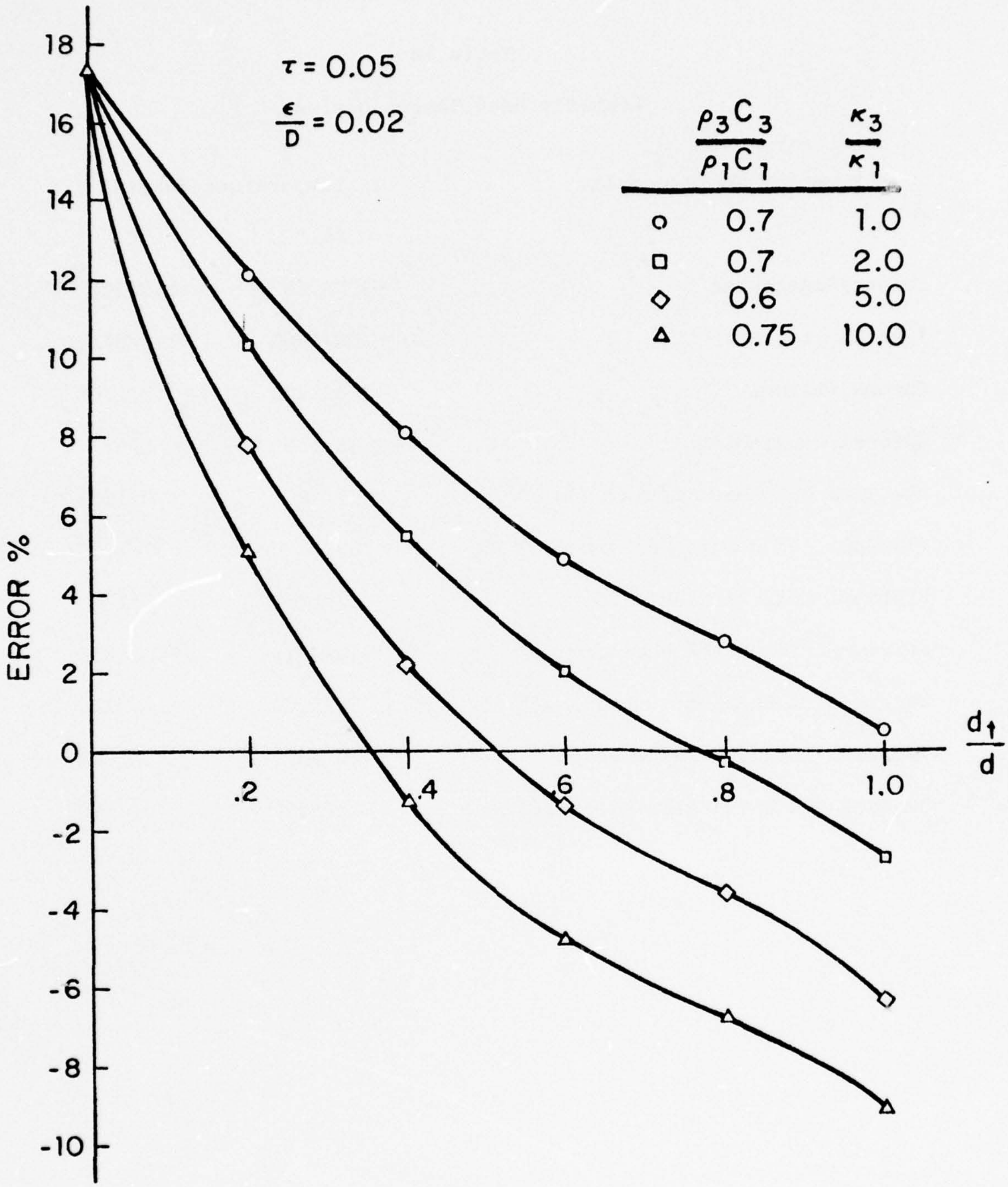

Figure 9 Percentage Error vs. d_t/d Ratio for Various κ_3/κ_1 and ρc Ratios $\varepsilon/D = 0.02$

Table 1a

Commonly Used Thermocouples

Types of Thermocouples	Temperature Ranges	
	°F	°C
Copper/Constantan	-300/750	-148/398
Iron/Constantan	-300/1600	-148/871
Chromel/Alumel	-300/2300	-148/1260
Chromel/Constantan	32/1800	0/982
Platinum 10% Rhodium/Platinum	32/2800	0/1537
Platinum 13% Rhodium/Platinum 6% Rh	100/3270	37/1798
Platinel 1813 Platinel 1503	32/2372	0/1300
Iridium/Iridium 60% Rhodium	2552/3326	1400/1830
Tungsten 3% Rhenium/Tungsten 25% Rhenium	50/4000	10/2204
Tungsten/Tungsten 26% Rhenium	60/5072	15/2800
Tungsten 5% Rhenium/Tungsten 26% Rhenium	32/5000	0/2760

Table 1b

Thermal Properties of Thermalcouple Materials

Metal, Insulator (subscript 3)	$K_3 \frac{cal}{sec.\ cm\,K}$	$\frac{\kappa_3}{\kappa\ steel}$	$\frac{\kappa_3}{\kappa\ aluminum}$	$\rho_3 c_3 \frac{cal}{cm^3}$	$\frac{\rho_3 c_3}{\rho c\ steel}$	$\frac{\rho_3 c_3}{\rho c\ aluminum}$
Aluminum	0.554	4.66	1.00	0.65	0.60	1.0
Copper	0.935	7.87	1.69	0.84	0.78	1.30
Chromium	0.201	1.70	0.36	0.84	0.78	1.30
Nickel	0.160	1.35	0.29	1.20	1.12	1.86
Platinum	0.174	1.46	0.31	0.74	0.69	1.14
Steel	0.119	1.0	0.21	1.08	1.00	1.66
Tungsten	0.373	3.14	0.67	0.59	0.55	0.92
Iridium	0.345	2.91	0.62	0.66	0.61	1.01
Rhodium	0.360	3.03	0.65	0.72	0.67	1.12
Rhenium	0.111	0.94	0.20	0.73	0.68	1.14
Nickel-Chromium	0.040	0.34	0.07	1.01	0.94	1.56
Copper-Nickel	0.059	0.50	0.11	0.99	0.92	1.52
Teflon	6×10^{-4}	0.005	0.001	0.54	0.5	0.83
Air	1.26×10^{-4}	0.001	0.0002	0.53	0.49	0.82

Note: The value quoted is the averaged value over 200 to 800 °K whenever data are available.

Table 2

Percentage Error of Temperature at Cavity Base (/D = 0.1)

Table 2.1 $\kappa_3/\kappa_1 = 0.5$ $\epsilon/D = 0.1$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.9	1	1.9	1.4	1.0	0.8	0.6	0.5
	2	3.4	2.5	1.9	1.4	1.1	1.0
	4	5.1	3.8	2.9	2.2	1.7	1.5
	8	6.4	4.7	3.7	2.9	2.3	1.9
	12	6.9	5.1	4.1	3.3	2.6	2.2
	20	7.4	5.5	4.5	3.7	2.9	2.4
	40	7.8	5.9	4.8	4.0	3.2	2.6
	60	7.9	6.0	4.9	4.0	3.2	2.6
	80	7.9	6.0	4.8	3.9	3.2	2.6
	100	7.9	6.0	4.8	3.9	3.2	2.6
1.3	1		1.3	0.9	0.6	0.4	0.3
	2		2.4	1.7	1.1	0.8	0.6
	4		3.7	2.6	1.8	1.2	1.1
	8		4.6	3.4	2.4	1.6	1.5
	12		5.0	3.8	2.7	1.9	1.8
	20		5.4	4.2	3.1	2.2	2.0
	40		5.8	4.5	3.4	2.5	2.3
	60		5.9	4.5	3.4	2.5	2.3
	80		5.9	4.5	3.3	2.5	2.3
	100		5.8	4.5	3.3	2.5	2.3
1.8	1		1.2	0.7	0.4	0.1	0.1
	2		2.3	1.4	0.8	0.4	0.3
	4		3.6	2.3	1.4	0.7	0.6
	8		4.5	3.0	1.9	0.9	1.0
	12		4.9	3.4	2.1	1.1	1.2
	20		5.3	3.8	2.5	1.5	1.6
	40		5.6	4.1	2.8	1.7	1.9
	60		5.7	4.1	2.7	1.7	1.9
	80		5.7	4.1	2.6	1.6	1.9
	100		5.7	4.0	2.6	1.6	1.9

Table 2.2 $\kappa_3/\kappa_1 = 1.0$ $\epsilon/D = 0.1$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.7	1		1.2	0.8	0.5	0.3	0.2
	2		2.3	1.5	1.0	0.6	0.2
	4		3.4	2.4	1.6	1.0	0.3
	8		4.3	3.1	2.1	1.3	0.3
	12		4.6	3.3	2.3	1.5	0.3
	20		4.9	3.6	2.6	1.6	0.3
	40		5.2	3.8	2.8	1.7	0.2
	60		5.2	3.9	2.8	1.7	0.2
	80		5.2	3.9	2.8	1.8	0.2
	100		5.2	3.9	2.8	1.8	0.2
0.95	1		1.2	0.7	0.4	0.2	0.0
	2		2.2	1.4	0.8	0.3	0.0
	4		3.3	2.2	1.2	0.6	0.0
	8		4.2	2.8	1.7	0.8	0.1
	12		4.5	3.1	1.9	0.9	0.0
	20		4.8	3.4	2.2	1.1	0.0
	40		5.1	3.6	2.3	1.2	0.0
	60		5.1	2.6	2.3	1.2	0.0
	80		5.1	3.6	2.3	1.2	0.0
	100		5.1	3.6	2.3	1.2	0.0
1.3	1		1.1	0.6	0.2	-0.0	-0.1
	2		2.1	1.1	0.5	-0.0	-0.2
	4		3.2	1.8	0.8	0.0	-0.3
	8		4.0	2.4	1.2	0.2	-0.3
	12		4.4	2.7	1.4	0.3	-0.3
	20		4.7	3.0	1.6	0.4	-0.3
	40		5.0	3.2	1.7	0.4	-0.2
	60		5.0	3.2	1.7	0.4	-0.2
	80		5.0	3.2	1.6	0.4	-0.2
	100		5.0	3.1	1.6	0.4	-0.2

Table 3

Percentage Error of Temperature at Cavity (/D = 0.04)

Table 2.3 $\kappa_3/\kappa_1 = 5.0$ $\epsilon/D = 0.1$

$\rho_3 c_3/\rho_1 c_1$	d_t/d → $\tau \times 10^2$ ↓	0.0	0.2	0.4	0.6	0.8	1.0
		Error %					
0.6	1		0.7	0.1	-0.3	-0.5	-1.0
	2		1.3	0.1	-0.4	-0.9	-1.8
	4		2.1	0.5	-0.6	-1.4	-3.0
	8		2.5	0.6	-0.7	-1.9	-4.5
	12		2.5	0.7	-0.7	-2.0	-5.4
	20		2.6	0.9	-0.4	-2.0	-6.4
	40		2.6	1.2	-0.1	-1.9	-7.2
	60		2.7	1.4	-0.2	-1.8	-7.3
	80		2.7	1.4	-0.2	-1.8	-7.3
	100		2.7	1.4	-0.2	-1.8	-7.3

Table 2.4 $\kappa_3/\kappa_1 = 10$ $\epsilon/D = 0.1$

$\rho_3 c_3/\rho_1 c_1$	$\tau \times 10^2$	0.0	0.2	0.4	0.6	0.8	1.0
0.75	1		0.3	-0.4	-0.8	-1.0	-1.4
	2		0.7	-0.6	-1.4	-1.9	-2.7
	4		1.1	-0.8	-2.0	-2.9	-4.6
	8		1.3	-0.9	-2.4	-3.7	-6.8
	12		1.3	-0.8	-2.3	-3.8	-8.3
	20		1.2	-0.4	-1.8	-3.7	-9.9
	40		1.4	0.2	-1.0	-3.3	-11.1
	60		1.4	0.3	-0.9	-3.2	-11.3
	80		1.4	0.4	-0.8	-3.2	-11.3
	100		1.4	0.4	-0.8	-3.2	-11.3

Table 3.1 $\kappa_3/\kappa_1 = 0.5$ $\epsilon/D = 0.04$

$\rho_3 c_3/\rho_1 c_1$	d_t/d → $\tau \times 10^2$ ↓	0.0	0.2	0.4	0.6	0.8	1.0
		Error %					
0.9	.2	1.5	0.9	0.7	0.5	0.4	0.4
	.4	3.2	2.0	1.5	1.1	0.8	0.7
	.8	5.6	3.9	2.8	2.0	1.5	1.3
	1.6	8.2	6.0	4.5	3.3	2.4	1.9
	2.4	9.3	7.0	5.3	3.9	2.9	2.3
	4.0	10.3	7.8	6.1	4.5	3.4	2.7
	8.0	11.2	8.5	6.7	5.2	3.9	3.1
	12.0	11.5	8.8	7.1	5.5	4.2	3.3
	16.0	11.8	9.0	7.3	5.8	4.5	3.4
	20.0	11.9	9.1	7.5	6.0	4.6	3.5
1.3	.2		0.8	0.5	0.3	0.2	0.2
	.4		1.9	1.2	0.8	0.5	0.4
	.8		3.7	2.5	1.6	1.0	0.8
	1.6		5.8	4.0	2.6	1.6	1.3
	2.4		6.8	4.9	3.2	2.1	1.6
	4.0		7.6	5.5	3.8	2.5	2.0
	8.0		8.4	6.3	4.4	2.9	2.5
	12.0		8.6	6.6	4.7	3.2	2.7
	16.0		8.8	6.9	5.0	3.4	2.9
	20.0		9.0	7.0	5.2	3.6	3.0
1.8	.2		0.7	0.4	0.2	0.1	0.0
	.4		1.7	0.9	0.5	0.2	0.1
	.8		3.4	2.1	1.1	0.5	0.3
	1.6		5.5	3.5	2.0	0.9	0.6
	2.4		6.5	4.3	2.5	1.2	0.9
	4.0		7.4	5.1	3.1	1.6	1.2
	8.0		8.2	5.8	3.6	1.9	1.7
	12.0		8.5	6.1	3.9	2.1	2.0
	16.0		8.7	6.3	4.1	2.3	2.2
	20.0		8.8	6.5	4.3	2.5	2.4

Table 3.2 $\kappa_3/\kappa_1 = 1.0$ $\varepsilon/D = 0.04$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.7	0.2		0.8	0.5	0.3	0.2	0.1
	0.4		1.8	1.1	0.7	0.4	0.3
	0.8		3.4	2.2	1.4	0.8	0.4
	1.6		5.4	3.6	2.2	1.3	0.4
	2.4		6.3	4.3	2.7	1.6	0.4
	4.0		7.1	5.0	3.2	1.9	0.4
	8.0		7.6	5.5	3.7	2.2	0.4
	12.0		7.9	5.8	4.0	2.4	0.4
	16.0		8.0	5.9	4.1	2.4	0.4
	20.0		8.1	6.1	4.2	2.5	0.3
0.95	0.2		0.7	0.4	0.2	0.1	0.0
	0.4		1.6	0.9	0.5	0.2	0.0
	0.8		3.3	1.9	1.0	0.4	0.1
	1.6		5.2	3.3	1.8	0.7	0.1
	2.4		6.1	3.9	2.2	0.9	0.1
	4.0		6.9	4.6	2.6	1.1	0.1
	8.0		7.5	5.1	3.1	1.4	0.1
	12.0		7.7	5.4	3.3	1.6	0.1
	16.0		7.9	5.6	3.5	1.6	0.1
	20.0		8.0	5.7	3.6	1.7	0.1
1.3	0.2		0.6	0.3	0.0	-0.1	-0.1
	0.4		1.5	0.7	0.2	-0.1	-0.2
	0.8		3.1	1.6	0.6	-0.1	-0.3
	1.6		5.0	2.8	1.2	0.1	-0.4
	2.4		5.9	3.5	1.5	0.2	-0.4
	4.0		6.7	4.1	1.0	0.3	-0.4
	8.0		7.3	4.6	2.2	0.4	-0.4
	12.0		7.6	4.9	2.5	0.5	-0.4
	16.0		7.7	5.1	2.7	0.6	-0.4
	20.0		7.9	5.2	2.8	0.7	-0.3

Table 3.4

Table 3.3 $\kappa_3/\kappa_1 = 5$ $\varepsilon/D = 0.04$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	.10
	$\tau \times 10^2$			Error %			
0.6	0.2		0.3	-0.1	-0.3	-0.4	-0.6
	0.4		0.9	0.0	-0.5	-0.8	-1.1
	0.8		2.0	0.4	-0.6	-1.2	-2.1
	1.6		3.4	0.9	-0.7	-1.8	-3.2
	2.4		4.1	1.2	-0.8	-2.1	-4.0
	4.0		4.6	1.4	-0.9	-2.5	-5.0
	8.0		4.7	1.4	-1.1	-3.0	-6.4
	12.0		4.7	1.4	-1.0	-3.2	-7.3
	16.0		4.7	1.6	-0.8	-3.2	-7.9
	20.0		4.7	1.7	-0.6	-3.2	-8.3

Table 3.4 $\kappa_3/\kappa_1 = 10$ $\varepsilon/D = 0.04$

0.75	0.2		0.0	-0.4	-0.6	-0.7	-0.9
	0.4		0.3	-0.6	-1.1	-1.4	-1.7
	0.8		1.0	-0.7	-1.7	-2.3	-3.0
	1.6		2.1	-0.8	-2.5	-3.5	-4.6
	2.4		2.6	-0.8	-2.9	-4.2	-5.7
	4.0		2.9	-0.9	-3.4	-5.0	-7.2
	8.0		2.9	-1.0	-3.7	-5.8	-9.5
	12.0		2.8	-0.8	-3.4	-5.8	-11.0
	16.0		2.8	-0.5	-3.1	-5.7	-11.9
	20.0		2.8	-0.3	-2.7	-5.6	-12.5

Table 4

Percentage Error of Temperature at Cavity Base (ϵ/D = 0.02)

Table 4.1 — κ_3/κ_1 = 0.5, ϵ/D = 0.02

Error %

ρ_3c_3/ρ_1c_1	$\tau \times 10^2$ \ d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
1.3	0.05		0.3	0.1	0.1	0.1	0.0
	0.10		0.8	0.4	0.2	0.2	0.1
	0.20		2.0	1.2	0.7	0.5	0.4
	0.40		4.5	2.7	1.6	1.0	0.8
	0.60		6.5	4.0	2.3	0.4	1.1
	1.00		8.9	5.6	3.3	2.0	1.5
	2.00		11.4	7.6	4.7	2.8	2.2
	3.00		12.4	8.5	5.4	3.3	2.6
	4.00		12.9	9.0	5.8	3.6	2.8
	5.00		13.1	9.3	6.0	3.7	3.0

Table 4.3 — κ_3/κ_1 = 10, ϵ/D = 0.02

ρ_3c_3/ρ_1c_1	$\tau \times 10^2$ \ d_t/d	0	0.2	0.4	0.6	0.8	1.0
0.75	0.05	0.7	0.0	-0.26	-0.33	-0.37	-0.41
	0.10	1.7	0.0	-0.48	-0.66	-0.76	-0.86
	0.20	4.9	0.2	-0.72	-0.93	-1.40	-1.61
	0.40	9.0	1.2	-0.85	-1.78	-2.28	-2.71
	0.60	11.6	2.0	-0.86	-2.20	-2.89	-2.50
	1.00	13.2	3.2	-0.84	-2.78	-3.76	-4.66
	2.00	15.8	4.5	-0.88	-3.66	-5.06	-6.44
	3.00	16.7	4.9	-0.99	-4.20	-5.84	-7.56
	4.00	17.1	5.1	-1.10	-4.57	-6.38	-8.42
	5.00	17.4	5.1	-1.20	-4.80	-6.75	-9.12

Table 4.2 — κ_3/κ_1 = 1, ϵ/D = 0.02

ρ_3c_3/ρ_1c_1	$\tau \times 10^2$ \ d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
0.7	0.05		0.3	0.2	0.1	0.1	0.1
	0.10		0.8	0.5	0.3	0.2	0.2
	0.20		1.9	1.1	0.7	0.4	0.3
	0.40		4.3	2.5	1.5	0.8	0.5
	0.60		6.1	3.6	2.1	1.1	0.5
	1.00		8.3	5.1	2.9	1.5	0.6
	2.00		10.6	6.8	3.9	2.1	0.6
	3.00		11.4	7.5	4.4	2.4	0.5
	4.00		11.8	7.8	4.7	2.5	0.5
	5.00		12.1	8.1	4.9	2.8	0.5
0.95	0.05		0.2	0.1	0.0	0.0	0.0
	0.10		0.7	0.3	0.1	0.1	0.0
	0.20		1.8	0.9	0.4	0.1	0.0
	0.40		4.0	2.1	1.0	0.3	0.1
	0.60		5.8	3.1	1.5	0.5	0.1
	1.00		8.0	4.5	2.2	0.8	0.1
	2.00		10.3	6.1	3.1	1.2	0.1
	3.00		11.2	6.8	3.5	1.4	0.1
	4.00		11.6	7.2	3.8	1.5	0.1
	5.00		11.8	7.4	4.0	1.7	0.1
1.3	0.05		0.2	0.0	-0.0	-0.1	-0.1
	0.10		0.5	0.1	-0.1	-0.1	-0.1
	0.20		1.6	0.6	0.1	-0.2	-0.3
	0.40		3.7	1.6	0.5	-0.2	-0.4
	0.60		5.4	2.5	0.8	-0.1	-0.5
	1.00		7.6	3.8	1.4	-0.0	-0.5
	2.00		10.0	5.4	2.1	0.2	-0.5
	3.00		10.9	6.0	2.5	0.3	-0.5
	4.00		11.3	6.4	2.7	0.4	-0.5
	5.00		11.5	6.7	2.9	0.4	-0.5

PART II IMPROVED ACCURACY IN THE PREDICTION OF SURFACE HEAT FLUX AND TEMPERATURE BY AN INTRINSIC THERMOCOUPLE

1. INTRODUCTION

In the study of transient heat transfer, many experimental difficulties may arise if heat flux sensors or thermocouples are installed direct at the surface of a body. For example, a probe may be damaged by a piston or a projectile sliding over a cylinder or barrel. A probe on a melting and ablative surface of heat shield can be easily destroyed because of high temperature. Furthermore a surface probe exposed to both radiative and convective environment may measure an erroneous surface heat flux and temperature if the probe has a different radiative property from that of the measured surface. In these circumstances, calculation of the transient surface heat flux and the surface temperature can be achieved by inverting a temperature history measured at some location inside the body.

In general, the prediction of a surface heat flux and temperature by the measured data at some location interior to a body is known as the "inverse problem". Many configurations, such as spheres, cylinders, and slabs, had been studied by many workers and many methods such as numerical, graphical, series, convolution integral, and Laplace transforms were used. Stolz [1], Beck [2] and Williams and Curry [3], considered the numerical inversion of the integral solution for semi-infinite and other bodies. In this method, care is required in selecting a time interval in order to achieve a stable solution. Carslaw and Jaeger [4], Burggraf [5], Koveryanov [6], and Shumakov [7], respectively considered different series approaches in which generally the local heat flux at an interior location and their higher derivaties are required. However, it is difficult to measure experimentally or to process the measured data for the derivative

of the temperature. Sparrow, Haji-Sheikh, and Lundgren [8], Imber and Kahn [9], Imber [10], Sabherwal [11], Masket and Vastano [12], Deverall and Channapragada [13] and Chen and Thomsen [14] applied the transform method. In these works, the solution is represented in either an integral form after some manipulation of the contour integral from the inverse transform, or in a series form after an expansion of the solution for small and large times. Using Laplace transformation Chen and Thomsen [14] introduced a polynomial in terms of an error function to represent the response of thermocouple measurement and the inversion is accomplished for any transient surface heat flux at the inner surface of a cylindrical tube. In thier study, the cylindrical thickness was assumed to be relatively thick such that the temperature at a large distance from the heating surface remains constant. Therefore, only one interior temperature response near the surface was needed in the experimental measurement. Their inversion solution however was valid only for a short duration due to the asymptotic expansion of the modified Bessel function in the inverse Laplace transform. Chen and Chiou [15] studied the inversion problem for the case of a semi-infinite slab or a thick slab using a Laplace transformation. The exact solution was obtained from the inverse Laplace transform for any time interval. It was then shown that their analysis may be approximately applied to the case of the hollow cylinder if the interior temperature response is measured at a location close to the inner wall.

This report presents (a) the improved numerical solution of the inversion solution reported by Chen and Chiou [15] and (b) a further demonstration of the capability of the solution.

The theoretical analysis of Chen and Chiou [15] is recapitulated in Appendix I in which the surface heat flux and temperature is predicted by inverting a temperature history measured at some location inside the solid body. The inversion solution is obtained by invoking Laplace transformation. Both the surface heat flux and temperature are given by Eqs. (19) and (20) in Appendix I.

It was thought that the accuracy of the computer program generated for the solution in the previous report by Chen and Chiou [15] can be improved further for the following reasons. First, the coefficients b_n (see Appendix Eq. (11)) in the previous formulation has a dimension of temperature. Therefore the determination of the coefficients depends on the temperature range of each particular experiment. It was found that the absolute value of the coefficients b_n in some cases can become as large as an order of 10^{44}. Therefore during the subsequently numerical manipulation in the computer program error due to round off and the standard fixed up when an underflow occurred may become appreciable. To remedy this difficulty the dimensionless formulation is introduced in the analysis (Appendix I) in which the coefficient b_n is also made dimensionless. As a result the magnitude of the coefficient b_n can be greatly reduced. Secondly, the double precision format was not used throughout the previous computer program. It is felt that further accurate results may be obtained if the double precision format is adopted in the program.

In Section II the new computer solution is shown to be indeed more accurate. Also in Section III the solution is shown to be capable of predicting a case involving a periodic surface heat flux or periodic temperature variation.

II. RESULTS OF THE IMPROVED COMPUTER PROGRAM

The previous computer program of Chen and Chiou [15] was recasted in dimensionless form and written in the double precision format. The new computer program is listed in Appendix II. The results predicted by the new and previous computer program are given in Appendix III and shown in Figure 1 for the case of the constant surface heat flux. This is the case in which a steel slab initially at a uniform temperature is suddenly subjected to a constant heat flux Q at one of the surfaces and kept at the initial temperature on the other surface. Figure 1 shows the solution predicted by inverting the temperature response at an interior of the slab from the new and previous computer program. This solution predicted by the new and previous programs used the ten term representation for the thermocouple response. The comparison clearly shows the improvement of the new solution over the previous one. Except for the short time duration the solution with the new program reduces the error to only one half of the error of the previous program i.e., an error of less than one percent. In the short time period the solution exhibits a Gibbs phenomenon* because of the discontinuity of the surface temperature gradient occurred at initial condition. The solution shows a 17% of initial overshot of heat flux and then a 7.8% of undershot before the solution approaches the constant heat flux. It should be remarked that Gibbs phenomenan is artificially

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* Gibbs phenomena [3]: for a sequence of transformation Tn(t), n = 1, 2,... of a function q(t) (here q(t) = constant) if the interval $\lim_{t \to t_o} \inf_{n \to \infty} Tn(t)$, $\lim_{t \to t_o} \sup_{n \to \infty} Tn(t)]$ contains points outside the interval $[\lim_{t \to t_o} \inf q(t)$, $\lim_{t \to t_o} \sup q(t)]$ then the sequence is said to exhibit a Gibbs phenomena.

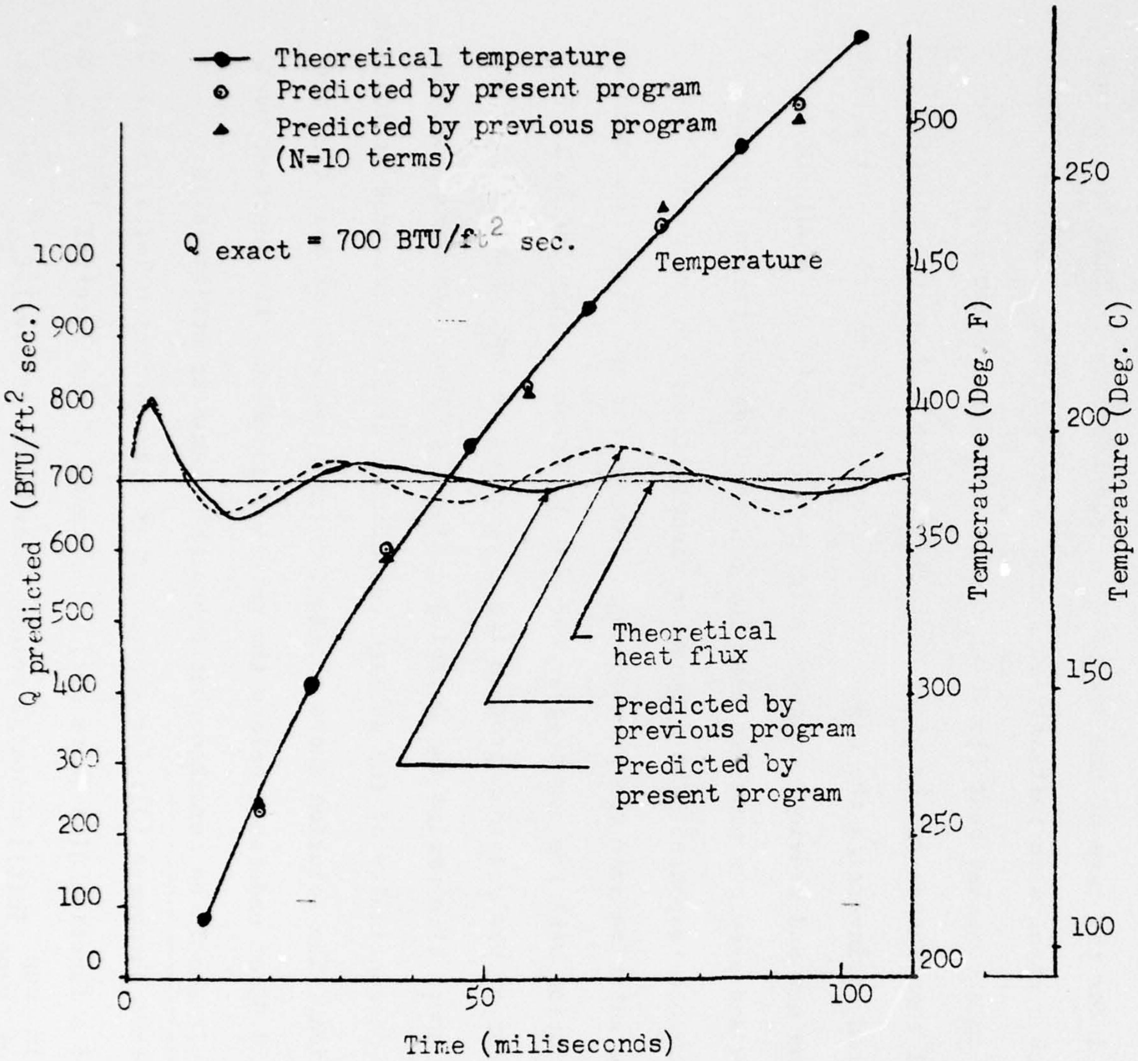


Figure 1 Comparison of Previous and present Programs

introduced due to the idealization of the initial condition. In most practical situations the surface heat flux will be continuous. Therefore, Gibbs phenomenon will not appear.

Figure 2 (see Table 1 also) shows the comparison between the solutions for the constant heat flux case with 10 and 20 term representation for the thermocouple response. One sees that the solution with 20 term representation after the initial Gibbs phenomenon quickly approaches the expected constant heat flux solution with a negligible error of less than 0.14 percent. This shows the accuracy of the new computer program. From Figure 2 one also observes that both overshoot and undershoot of Gibbs phenomenon are smaller for the 20 term representation. Additionally the points of the overshoot and the undershoot has moved to near the zero time which agrees with the characteristic of Gibbs phenomenon. According to Gibbs phenomenon the point of overshoot shoot should approach the initial zero if the number of term of the series which represent the thermocouple response is increased to infinite.

VII. VERIFICATION OF OSCILLATORY SOLUTION

As a measure of applicability of the present inversion solution, a test problem was solved for the case of a slab subjected to a preiodic surface temperature variation on one surface and held to the initial temperature on the other. The analytic solution for the problem is given in Appendix IV where a more suitable form of the solution than the one given by Carslaw and Jaeger [4] is derived and tabulated for the thermocouple response at one tenth of the slab thickness from the surface. The

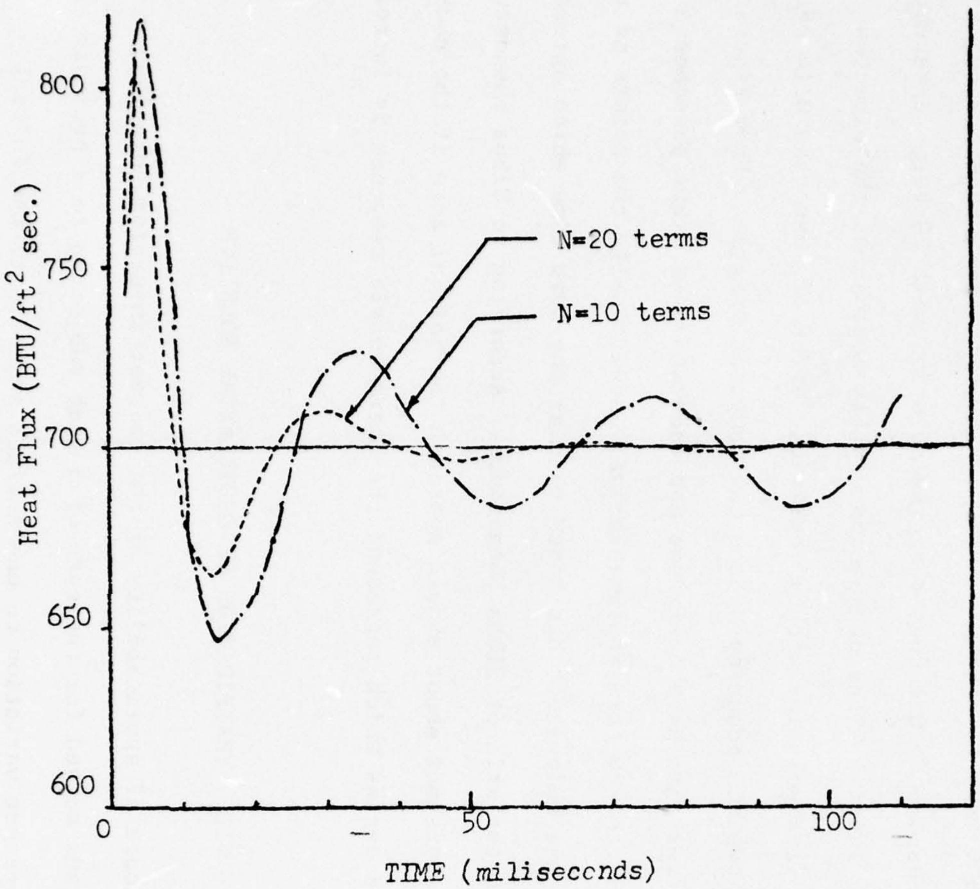


Figure 2 Comparison of Different Polynomial Representations

TABLE 1

Comparision of Inversion Prediction and Exact Solution

N	t	$\frac{f(t)}{\theta(1,t)}$	$\theta(0,t)$ EXACT	$\theta(0,t)$ PREDICTED	ERROR* %	$\frac{\partial\theta}{\partial x}(0,t)$ EXACT	$\frac{\partial\theta}{\partial x}(0,t)$ PREDICTED	ERROR* %
20	0.1	0.0624	0.3758	0.3773	+0.399	1.0000	0.9873	-1.27
	0.2	0.1628	0.5315	0.5314	+0.019	1.0000	1.0014	+0.14
	0.4	0.3395	0.7516	0.7516	0.00	1.0000	1.0001	+0.01
	0.6	0.4882	0.9205	0.9205	0.00	1.0000	1.0000	0.00
	0.8	0.6183	1.0629	1.0628	-0.009	1.0000	0.9996	-0.04
	1.0	0.7353	1.1884	1.1878	-0.05	1.0000	1.9986	-0.14
10	0.1	0.0624	0.3758	0.4048	+7.71	1.0000	1.0796	+7.96
	0.2	0.1628	0.5315	0.5284	-0.58	1.0000	0.9391	-6.09
	0.4	0.3395	0.7516	0.7516	0.00	1.0000	1.0179	+1.79
	0.6	0.4882	0.9205	0.9205	0.00	1.0000	0.9923	-0.77
	0.8	0.6183	1.0629	1.0626	-0.028	1.0000	1.0043	+0.43
	1.0	0.7353	1.1884	1.1887	+0.025	1.0000	0.9963	-0.37

*ERROR % = ((PREDICTED)-(EXACT))/(EXACT)

surface is subjected to a periodic temperature variation with a period of 8 milliseceonds. Fifteen data points of the temperature response at the thermocouple location are then input to the inversion program for prediction of the surface temperature and heat flux. The result is shown in Figure 3 and Appendix III where the data symbol "2" denotes the thermocouple response and "1" the surface temperature. The accuracy of the inversion program is shown in Table 2. Except for the extremely short time period of 0.4 milliseconds the prediction by the inversion program with 15 term representation is within 2 percent of error.

The accuracy can be improved more if more data points are used. Figure 4 shows the predicted surface heat flux which we were unable to compute from the series solution (Eq. (5) of Appendix IV). This demonstrates the versatility of the inversion solution.

IV. APPLICATION OF THE INVERSION PROGRAM

Three sets of data (see Appendix III) provided by Rock Island Arsenal for the temperature response of a thermocouple embedded in a M60 gun barrel were utilized to evaluate the inversion solution. The inversion prediction for the surface heat flux from all three sets of data were extremely high when compared with other known data calculated by Chen and Chiou [15]. Since the program correctly predicted the surface heat flux for other sets of experimental data it was judged that the three sets of data may contain inaccurate initial time. For most experimentations the recording instrument is likely to experience some delay in responding to the extremely fast transient heat flux typical in gun bores. Therefore an advanced shift of time of 2 milliseconds in the data was tested. The

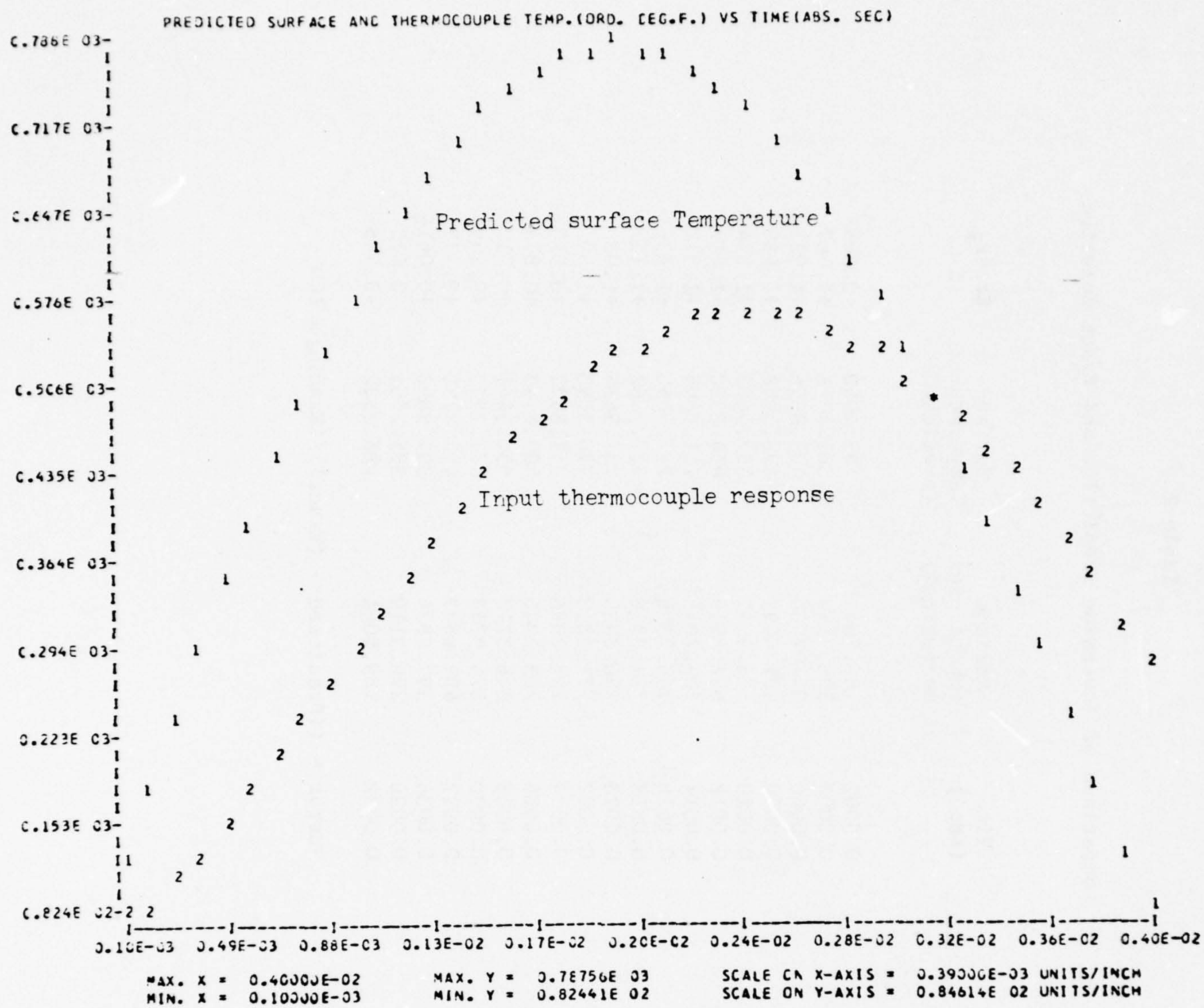


Figure 3 Predicted Surface Temperature

Table 2

Comparison of Inversion Prediction and Exact Solution

Time (sec.)	Surface Temperature (theoretical)	Surface Temperature (predicted)	Error* %
0.0002	189.5041	193.4060	+3.563
0.0004	296.3119	301.4311	+2.367
0.0006	397.7934	403.8998	+1.921
0.0008	491.4497	498.3988	+1.689
0.0010	574.9747	582.5683	+1.534
0.0012	646.3119	654.3358	+1.417
0.0014	703.7046	711.9396	+1.321
0.0016	745.7396	753.9650	+1.236
0.0018	771.3818	779.3802	+1.157
0.0020	780.0000	787.5615	+1.08
0.0022	771.3818	778.3093	+1.00
0.0024	745.7396	751.8527	+0.918
0.0026	703.7046	408.8443	+0.874
0.0028	646.3119	650.3441	+0.712
0.0030	574.9747	577.7933	+0.569
0.0032	491.6697	492.9790	+0.372
0.0034	397.7934	397.9893	+0.0616
0.0036	296.3119	295.2761	-0.679
0.0038	189.5041	188.8522	-0.595

*Error = ((Predicted - (Exact))/(Exact)) x 100

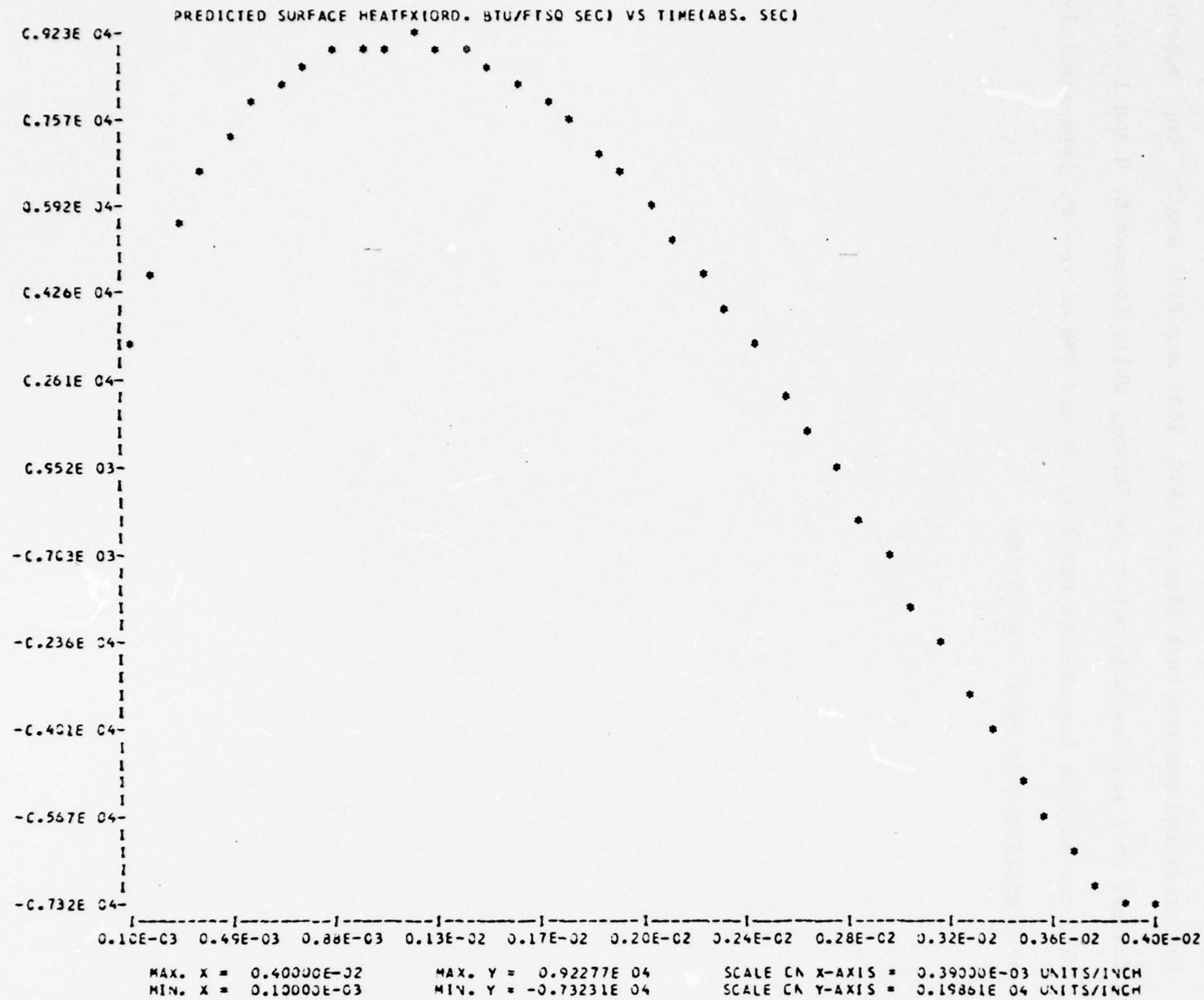
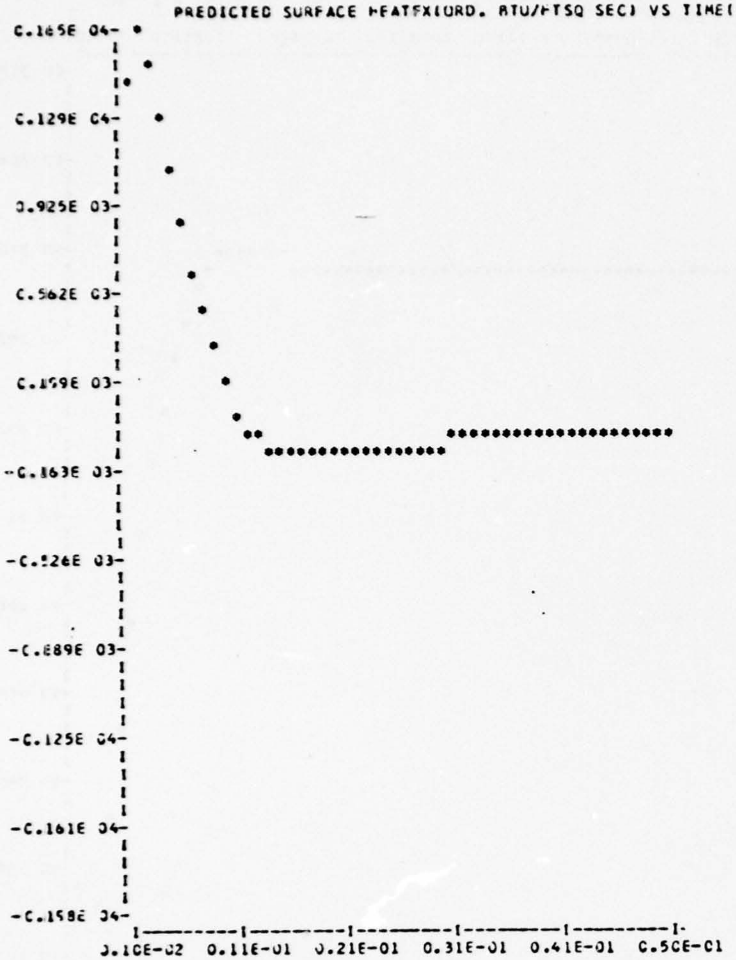


Figure 4 Predicted Surface Heat Flux

result is shown in Appendix III-3. Figures 5, 6 and 7 respectively show the predicted maximum heat flux of 1650, 1235 and 2695 Btu/ft^2 sec. approximately at 2 milliseconds after the firing, while Figures 8, 9 and 10 show that the surface temperature are 273, 234 and 396 degree F. approximately at 6 milliseconds after the firing.

Figure 5 M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREECH

PREDICTED SURFACE HEATFX(URD. BTU/FTSQ SEC) VS TIME(ABS. SEC)

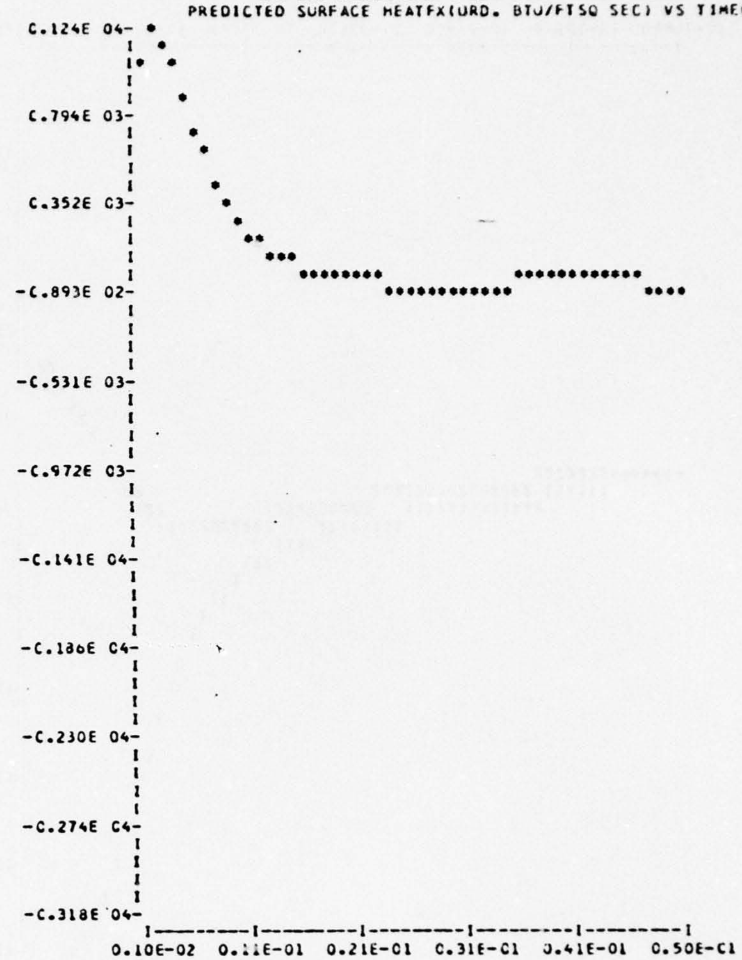


MAX. X = 0.10000E 00 MAX. Y = 0.16504E 04
MIN. X = 0.10000E-02 MIN. Y = -0.19771E 04

SCALE ON X-AXIS = 0.99000E-02 UNITS/INCH
SCALE ON Y-AXIS = 0.43530E 03 UNITS/INCH

Figure 6 M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH

PREDICTED SURFACE HEATFX(URD. BTU/FTSQ SEC) VS TIME(ABS. SEC)

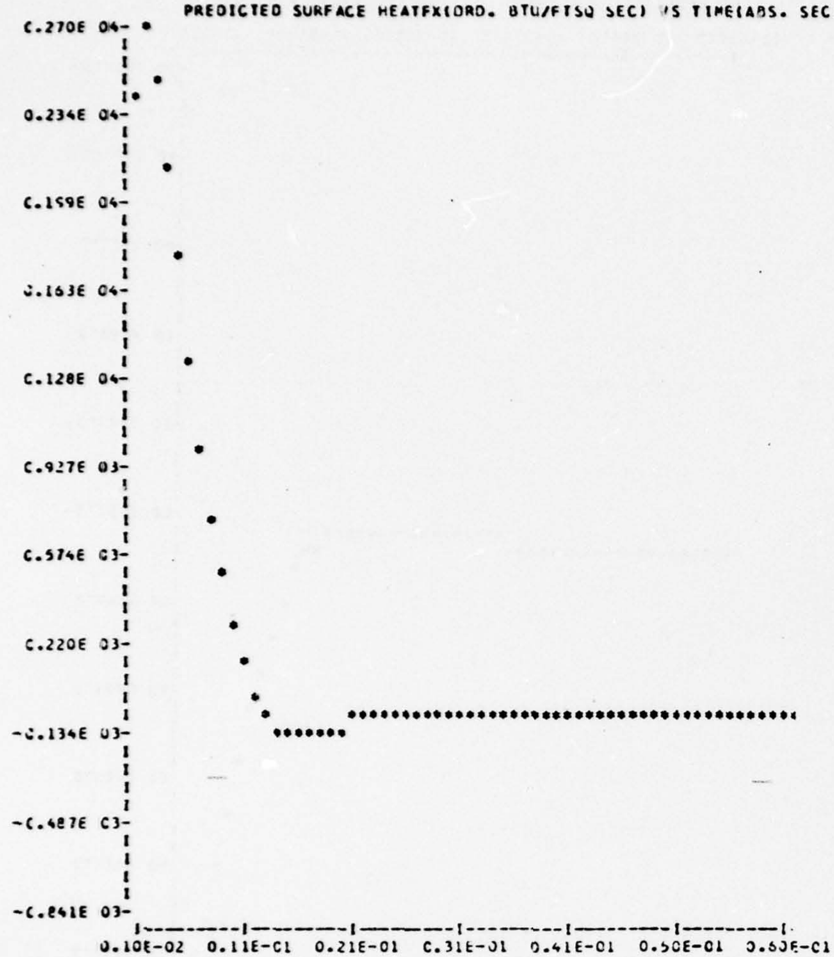


MAX. X = 0.10000E 00 MAX. Y = 0.12354E 04
MIN. X = 0.10000E-02 MIN. Y = -0.31802E 04

SCALE ON X-AXIS = 0.99 0E-02 UNITS/INCH
SCALE ON Y-AXIS = 0.52 7E 03 UNITS/INCH

Figure 7 M60 GUN THERMOCOUPLE 4 9.0 INCHES FROM BREECH

PREDICTED SURFACE HEATFX(ORD. BTU/FTSQ SEC) VS TIME(ABS. SEC)

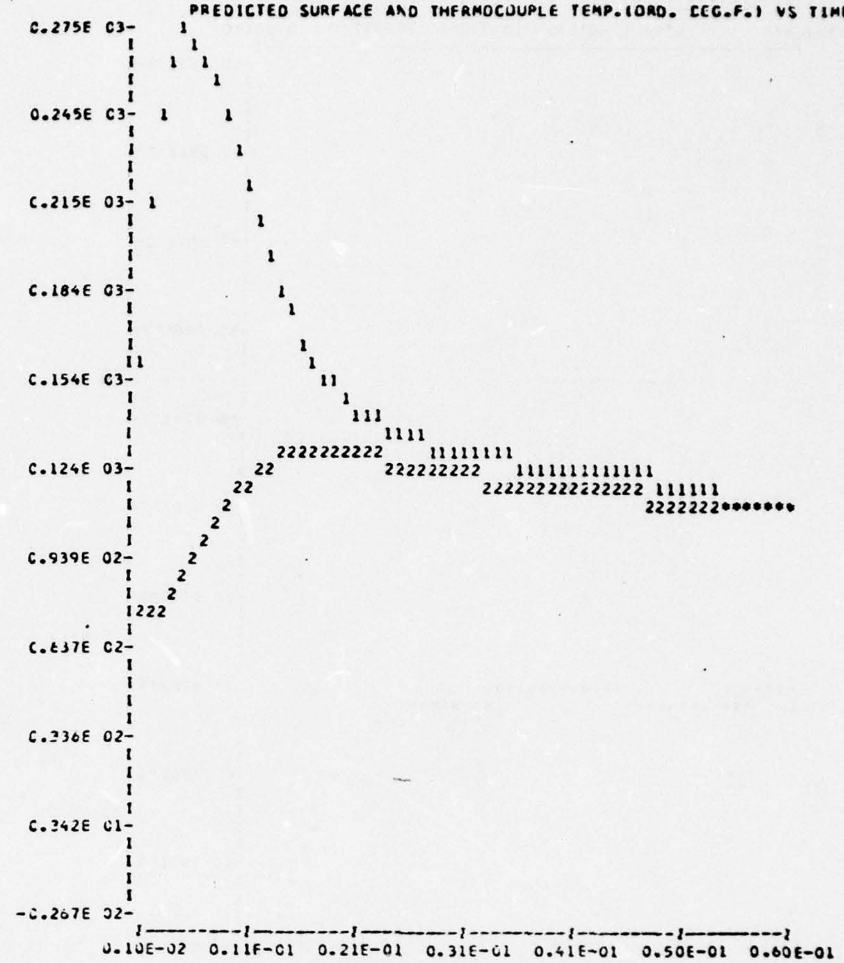


MAX. X = 0.10000E 00   MAX. Y = 0.26952E 04
MIN. X = 0.10000E-02   MIN. Y = -0.94083E 03

SCALE ON X-AXIS = 0.99000E-02 UNITS/INCH
SCALE ON Y-AXIS = 0.42433E 03 UNITS/INCH

Figure 8 M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREECH

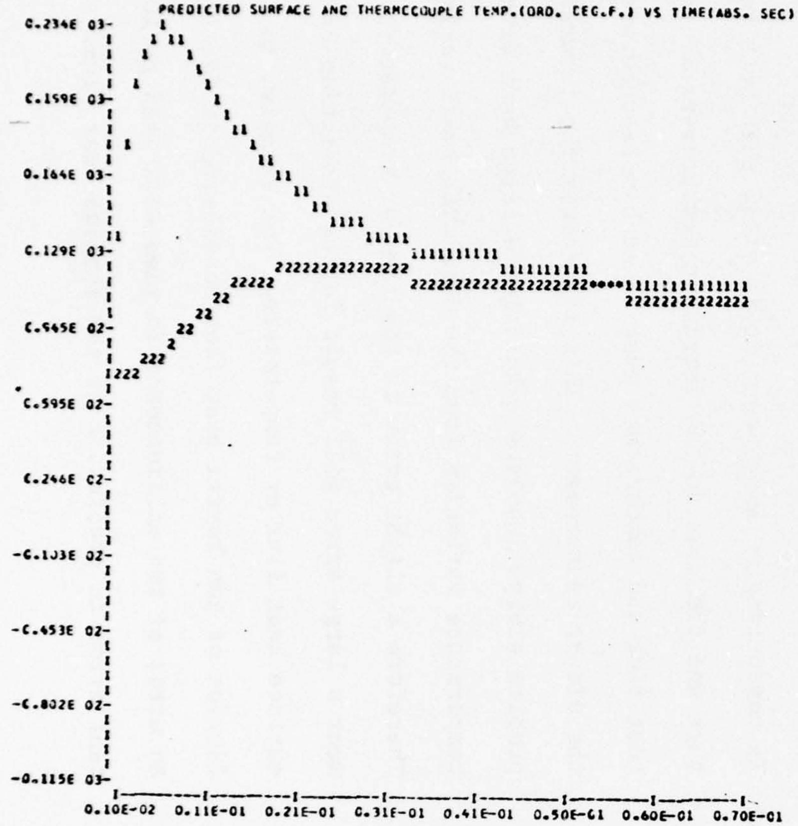
PREDICTED SURFACE AND THERMOCOUPLE TEMP.(ORD. DEG.F.) VS TIME(ABS. SEC)



MAX. X = 0.10000E 00   MAX. Y = 0.27482E 03
MIN. X = 0.10000E-02   MIN. Y = -0.26731E 02

SCALE ON X-AXIS = 0.99000E-02 UNITS/INCH
SCALE ON Y-AXIS = 0.30186E 02 UNITS/INCH

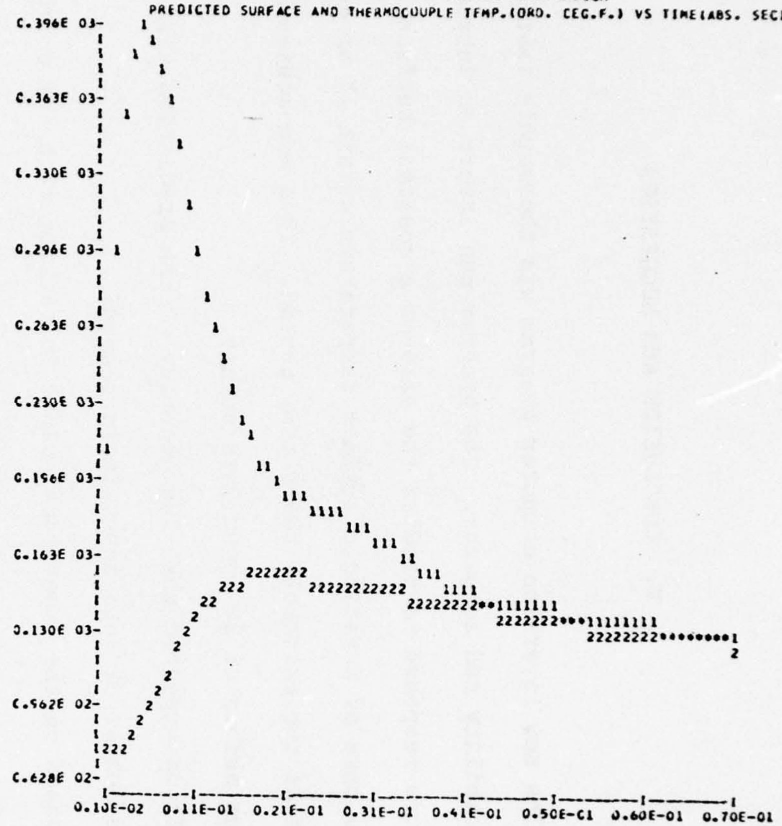
Figure 9 M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH
PREDICTED SURFACE AND THERMOCOUPLE TEMP.(ORD. DEG.F.) VS TIME(ABS. SEC)



MAX. X = 0.10000E 00 MAX. Y = 0.23425E 03
MIN. X = 0.10000E-02 MIN. Y = -0.11517E 03

SCALE ON X-AXIS = 0.99000E-02 UNITS/INCH
SCALE ON Y-AXIS = 0.41130E 02 UNITS/INCH

Figure 10 M60 GUN THERMOCOUPLE 4 9.0 INCHES FROM BREECH
PREDICTED SURFACE AND THERMOCOUPLE TEMP.(ORD. DEG.F.) VS TIME(ABS. SEC)



MAX. X = 0.10000E 00 MAX. Y = 0.39641E 03
MIN. X = 0.10000E-02 MIN. Y = 0.62848E 02

SCALE ON X-AXIS = 0.99000E-02 UNITS/INCH
SCALE ON Y-AXIS = 0.40028E 02 UNITS/INCH

V. CONCLUSION AND SUGGESTION

The new inversion computer program was thoroughly tested for its applicability and accuracy. The program can invert an intrinsic temperature response to predict the case of a constant surface heat flux or the case of a periodic surface temperature within 2% of deviation except at the extremely short time period. This was achieved with the maximum number of 20 input data points.

It is expected that the accuracy of the prediction will increase if the number of data input is increased.

Based on the experience gained in working with the program the following suggestions are thought relevant.

(1) In conducting an experiment it is vital that both temperature and the time in the intrinsic measurement of surface heat flux and temperature must be much more accurate than the direct measurement. This is because the inversion problem always involves predicting a large heat flux or temperature variation from the data with small variation. Therefore a slight error in the time or temperature measurement a large error will result in the prediction of the surface heat flux or temperature. For example, in the prediction of gun barrel heat flux considered in Section III an error of two milliseconds in time will lead to 100 percent error in prediction of the surface heat flux.

(2) In selecting the data points for input to the computer program care must be excericised not to create a locally abrupt jump in the data. An abrupt change in data points will often introduce an abnormal fitting of a curve in its neighborhood and hence resulting in an incorrect prediction of the surface heat flux and temperature. If indeed the abrupt jump of the data must be used, then more data points in its neighborhood must also be chosen.

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APPENDIX I. ANALYSIS OF THE INVERSION PROBLEM

Consider a slab, having a sufficient wall thickness, L, such that the outer surface temperature has a negligible response when the inner surface is exposed to a transient heat flux. A probe, for example a thermocouple, is located at $X = X_1$ and it is normally desirable, as reported by Chen and Li [16], to be close to the heating surface since a better transient response and more accurate experimental measurements can be obtained to reduce error amplification in the mathematical inversion program. Under these circumstances we thus assume in the analysis $L/X_1 >> 1$. The governing equation for the transient heat conduction may be written in a dimensionless form as

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} \tag{1}$$

with the initial and boundary conditions

$$\theta(x, o) = 0 \tag{2}$$

$$\theta(\infty, t) = 0 \tag{3}$$

$$\theta(1, t) = f(t) \tag{4}$$

Where $x = X/X_1$ is a dimensionless distance from the heating surface and $t = \alpha\tau/X_1^2$ is a dimensionless time or Fourier number with α being the thermal diffusivity and τ the real time. $\theta = (T - T_o)/T_o$ is the dimensionless temperature above the initially uniform temperature T_o. $f(t) = (F(\tau) - T_o)/T_o$ is the dimensionless measured temperature response at $x = 1$ with $F(\tau)$ being the measured absolute temperature. The

inversion problem is then given the interior temperature $f(t)$ to predict the surface temperature $\theta(0,t)$ and the surface heat flux per unit area $q(0,t)$ or $\left.\frac{\partial\theta}{\partial x}\right|_o = -q(0,t)X_1/T_o\kappa$. Here κ is the thermal conductivity of the solid.

The above problem may be solved by Laplace Transformation. Let the transformation be:

$$\bar{\theta}(x,s) = \int_o^\infty \theta(x,t)e^{-ts}\,dt \tag{5}$$

where θ is continuous otherwise satisfied the Dirichlet's condition. The temperature function θ is recovered by inversion of the Laplace Transformation as:

$$\theta(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \bar{\theta}e^{st}\,ds \tag{6}$$

where c is a suitable positive value. Equation (1) and the initial condition (2) under transformation (5) become:

$$\frac{d^2\bar{\theta}}{dx^2} = s\bar{\theta} \tag{7}$$

which has a solution, with integration constants A and B,

$$\bar{\theta}(x,s) = A\,e^{\sqrt{s}x} + Be^{-\sqrt{s}x} \tag{8}$$

The transformation of boundary conditions (3) and (4) into the Laplace

plane give $\bar{\theta}(\infty,s) = 0$, $\bar{\theta}(1,s) = \bar{f}(s)$. Substitution of the boundary conditions into equation (8), we get $\bar{\theta}$ and its derivative as:

$$\bar{\theta}(x,s) = \bar{f}(s)\, e^{\sqrt{s}(1 - x)} \tag{9}$$

$$-\frac{\partial\bar{\theta}}{\partial x} = \sqrt{s}\ \bar{f}(s)\, e^{\sqrt{s}(1 - x)} \tag{10}$$

According to Chen and Thomsen [14], we choose the temperature response as measured by a probe to be represented by a polynomial

$$f(t) = \sum_{n=1}^{N} b_n\, (4t)^n\, \Gamma(n+1)\, i^{2n} \operatorname{erfc}\left(\frac{1}{2\sqrt{t}}\right) \tag{11}$$

or in Laplace plane

$$\bar{f}(s) = e^{-\sqrt{s}} \sum_{n=1}^{N} \Gamma(n + 1)\, \frac{b_n}{s^{1+n}} \tag{12}$$

The b_n's are coefficients of the expansion to be determined such that the N term polynomial describes the temperature response $f(t)$ measured at $x = 1$. With equation (12), equations (9) and (10) can be simplified to

$$\bar{\theta}(x,s) = e^{-x\sqrt{s}} \sum_{n=1}^{N} \Gamma(n + 1)\, \frac{b_n}{s^{1+n}} \tag{13}$$

$$-\frac{\partial\bar{\theta}}{\partial x} = e^{-x\sqrt{s}} \sum_{n=1}^{N} \Gamma(n + 1)\, \frac{b_n}{s^{1/2+n}} \tag{14}$$

The inversion of equations (13) and (14) at $x = 0$ give:

$$\theta(0,t) = \sum_{n=1}^{N} b_n t^n \tag{15}$$

$$-\left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=0} = \sum_{n=1}^{N} b_n t^{n-1/2} \frac{\Gamma(n+1)}{\Gamma(n+1/2)} \tag{16}$$

where $\theta(0,t)$ gives the surface temperature and $-\frac{\partial\theta(0,t)}{\partial x}$ gives the surface heat flux $\frac{qX_1}{\kappa T_o}$ as a function of time.

The integral of the error function in (11) is defined as:

$$i^{2n} \operatorname{erfc}\left(\frac{1}{2\sqrt{t}}\right) = \int_{\left(\frac{1}{2\sqrt{t}}\right)}^{\infty} i^{2n-1} \operatorname{erfc}(y)\, dy \tag{17}$$

with $n = 0$, $\operatorname{erfc}(y) = \frac{2}{\sqrt{\pi}} \int_{y}^{\infty} e^{-x^2} dx$. $\Gamma(n)$ in equations (11) and (16) is the gamma function or *Euler's integral function of the second kind.*

$$\Gamma(n) = \int_{0}^{\infty} e^{-\omega} \omega^{n-1} d\omega \tag{18}$$

It should be remarked that the choice of the particular form (11) is to ensure the convergence of the solution on the Laplace plane and an analytic inversion back to the physical plane. With b_n coefficients determined from equation (11) and the experimental measurement of the temperature response $f(t)$ at $x = 1$, the surface temperature, $T_w(t)$, is obtained from equation (15) as

$$T_w(t) = T_o\left(1 + \sum_{n=1}^{N} b_n t^n\right) \tag{19}$$

and the heat flux, $q(t) = \frac{-T_o \kappa}{X_1} \left. \frac{\partial \theta}{\partial x} \right|_{x=0}$, from equation (16) as:

$$q(t) = \frac{-T_o \kappa}{X_1} \sum_{n=1}^{N} b_n \, t^{n-1/2} \frac{\Gamma(n+1)}{\Gamma(n+1/2)} \tag{20}$$

The above solution is the exact solution for predicting the transient surface heat flux and temperature valid for both short and long time durations as long as the slab is thick enough such that the outer surface maintains its initial temperature. The feature of the present solution is the polynomial (11) which on the Laplace plane gives a term in equation (12) $\exp(-\sqrt{s})$ to cancel the term $\exp(\sqrt{s})$ in equations (9) and (10). This polynomial (11) as suggested by Chen and Thomsen [15] makes the present solution simpler than many inversion solutions derived in the past and valid in any time.

If the fluid temperature, $T_g(t)$, away from the surface of the slab is known, then the instantaneous heat transfer coefficients, $h(t)$, can be determined from Newton's cooling law as:

$$h(t) = \frac{q(t)}{T_g(t) - T_w(t)} \tag{21}$$

where heat flux, $q(t)$, and wall temperature $T_w(t)$ are given by equations (19) and (20), and the average heat transfer coefficient up to time t, $\bar{h}(t)$, can be defined as:

$$\bar{h}(t) = \frac{\int_0^t q(t')\, dt'}{\int_0^t [T_g(t') - T_w(t')]\, dt'} \tag{22}$$

APPENDIX II

IMPROVED COMPUTER PROGRAM

Cartesian Inversion Problem

```
FORTRAN IV G LEVEL  21              MAIN              DATE = 76157        15/33/00          PAGE 0001
                C I      ** DATA INPUT **                                                     I
                C I      ================                                                     I
                C I                                                                           I
                C I            TITLE OF DATA(BLANCK FIRST 20 SPACE)          FIRST CARD  I
                C I            BORRAD=BORE RADIUS OF CYLINDER (FT.)          SECOND CARD I
                C I            OUTR= OUTER RADIUS (FT.)                      THIRC CARD  I
                C I            THICK=OUTER RADIUS-BORRED (FT.)                           I
                C I            DIS=BORE TO THERMOCOUPLE DISTANCE (FT.)       FOURTH CARD I
                C I            TEMPO=INITIAL THERMOCOUPLE TEMPERATURE(F.)    FIFTH  CARD I
                C I            TG(1)= INITAL GAS CORE TEMPERATURE AT NORM TIME 0.00001   I
                C I                                                          SIXTH CARD  I
                C I            ALP=THERMAL DIFFUSIVITY OF MATERIAL(FTSC/SEC)SEVENTH CARDI
                C I            THCON=THERMAL CONDUCTIVITY OF MATERIAL(BTU/FT.SEC.DEG.F)  I
                C I                                                          EIGHTH CARD I
                C I            NP=NUMBER OF TIME-TEMPERATURE DATA  PAIRS UP TO 25 PAIRS  I
                C I                                                          NINTH CARD  I
                C I            NB= NUMBER OF B COEFF. TO BE FITTED           TENTH CARD  I
                C I            FROM 11TH  CN AR PAIRS OF TIME(I) AND TEMP(I)             I
                C I            TIME(I)=TIME FROM  THERMOCOUPLE RESPCASE CURVE(SEC.)      I
                C I            TEMP(I)= TEMPERATURE FROM THERMOCOUPLE RESPONSE (DEG. F)  I
                C I            TSHFT = TIME SHIFTED FROM DATA READ IN(SEC)   LAST CARD   I
                C I______________________________________________________________________I
                C
                C
                C ________________________________________________________________________
                C I            DECLARATICN STATEMENTS   START COMPUTER PROGRAM            I
                C I______________________________________________________________________I
                C
0001                  IMPLICIT REAL*8(A-H,O-Z)
0002                  DIMENSION        TG(210),QM(210),DTWM(210),HM(210),TGM(210),
                     1TWM(210),TOT(210),TEMO(210),HT(210),EQUR(210),DTEMR(210)
0003                  DIMENSICN B(26),BERFC(52),TEMP(26),TIME(26),T(26),X(26),GAM(28),
                     1C(26,26),FN(26),         G(26,210),H(26,210),ECUE(210),DTEMP(210)
                     2,TCT(26),TEMC(210),TRAN(26,26),QTQ(26,26)
0004                  DATA EPS/.1D-75/
                C
                C
                C ________________________________________________________________________
                C I            READ DATA CARDS INTO PROGRAM                               I
                C I______________________________________________________________________I
                C
0005                  CALL ERRSET(208,0,-1,1,1)
0006            199   FORMAT(    20X,50H
                     1                 )
0007            200   FORMAT(F20.10)
0008            201   FORMAT(I2)
0009            1     READ (5,199,END=999)                                            FIRST
0010                  READ (5,200) BORRAD                                             SECOND
0011                  READ (5,200) OUTR                                               THIRD
0012                  READ(5,200) DIS                                                 FOURTH
0013                  READ(5,200) TEMPO                                               FIFTH
0014                  READ (5,200) TG(1)                                              SISTH
0015                  READ(5,200) ALP                                                 SEVENTH
0016                  READ(5,200) THCON                                               EIGHTH
0017             2    READ(5,201, END=999) NP                                         NINTH
0018             3    READ (5,201) NB                                                 TENTH
0019                  WRITE (6,198)
0020            198   FORMAT (1H1)
0021                  WRITE (6,199)
```

```
            WRITE(6,100)
      100   FORMAT ( 20X,' BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM')
            WRITE (6,109) NB
               DC 10 I=1,NP
               READ(5,202) TIME(I),TEMP(I)
       10      CONTINUE
            WRITE(6,107)
            DO 16 I=1,NP
       16      WRITE(6,108) TIME(I),TEMP(I)
      202      FORMAT(2F20.10)
            READ (5,200)  TSHFT
            WRITE (6,130) TSHFT
C
C ------------------------------------------------------------------
C I            NOW THAT THE DATA HAS BEEN INPUT START CALCULATIONS  I
C I       FIRST SETUP DATA NEEDED FOR THE CALL TO PERFC             I
C I       WHERE PERFC IS INTEGRATED ERROR FUNCTION                  I
C I_________________________________________________________________I
C
                        DO 5 I=1,NP
        5               TIME(I) = TIME(I) - TSHFT
            THICK=CUTR-BORRAD
            RO=0.5D0
            GI=1.0D0
            TR1=ALP/(DIS**2)
C
C I ----------------------------------------------------------------I
C I            CACULATE DIMENSIONLESS TIME INCREMENT  'TEMIN'       I
C I       THE CONSTANT CRLT VARIES WITH WHAT REAL TIME RANGE WE WANT I
C I       BUT NOT DEPENDS CN DIFFERENT MATERIAL OR THERMOCOUPLE     I
C I       LOCATION                                                  I
C I ----------------------------------------------------------------I
C
            CRLT=0.1D-2
            TEMIN=TR1*CRLT
            XXI=0.5D0*GI
            WRITE (6,203) GI,TR1
      203   FORMAT ('-FOR CHECKING USE  GI=',1F7.4 ,'  TR1=',1F7.4 )
            DO 20 I=1,NP
               T(I)=TIME(I)*TR1
       20   X(I)=0.5D0/DSQRT(T(I))
       21   CONTINUE
C
C
C ------------------------------------------------------------------
C I       USE THE SUBROUTINE PERFC TO CALCULATE INTEGRATED ERROR    I
C I       FUNCTION AND SET UP  C(I,J) MATRIX FOR THE SOLUTION OF B  I
C I_________________________________________________________________I
C
            NN1 =NB+1
      204   FORMAT ('-NN1 = ', I2)
            WRITE (6,204)  NN1
            DC 30 I=1,NP
       30      C(I,NN1)=(TEMP(I)-TEMPO )/(460.0D0+TEMPO)
            DO 40 I=1,NP
            NB2=NB*2.0D0
            XI = X(I)
            CALL PERFC (NB2,BERFC,XI)
```

```
0058                DO 40 J=1,NB
0059             J2=2*J
0060             C(I,J)=(4.0D0*T(I))**J*BERFC(J2)*DGAMMA(J+1.0D0)
0061       40    CONTINUE
            C
            C ___________________________________________________________________
            C I      NOW B COEFFICIENT FOR THE THE TEMPERATURE RESPONSE AT        I
            C I      THE THERMOCOUPLE ARE SOLVED EITHER BY EXACT OR LEAST SQAURE  I
            C I__________________________________________________________________I
            C
0062             NT=NP+1
0063             IF (NP - NB) 42, 43, 42
0064       42    CALL LTSQER (NP,NB,C,B,EPS,TIME,NT,TRAN,QTQ,TEMPO)
0065             GO TO 44
0066       43    DET = SIMUL (NP, C, B, EPS, 1,11)
0067       44    CONTINUE
            C
            C ___________________________________________________________________
            C I           NOW CALCULATE HEAT FLUX FUNCTION  AND                  I
            C I           CALCULATE TEMPERATURE FUNCTION                         I
            C I__________________________________________________________________I
            C
0068             TEM = TEMIN
0069             DO 55 I=1,200
0070                DO 50 J=1,NB
0071             G(J,I)= TEM**J
0072             H(J,I)=-TEM**(J-0.5D0)*DGAMMA(J+1.0D0)/DGAMMA(J+0.5D0)
0073       50          CONTINUE
0074             TEM = TEM + TEMIN
0075       55       CONTINUE
            C
            C
            C ___________________________________________________________________
            C I           CALCULATE TEMPERATURE AND HEAT FLUX                    I
            C I__________________________________________________________________I
            C
0076             DO 75 I=1,200
0077                DELT=0.0D0
0078                QUE=0.0D0
0079                DO 70 J=1,NB
0080                   DELT=DELT+B(J)*G(J,I)
0081       70          QUE=QUE+B(J)*H(J,I)
0082                DTEMP(I)=DELT
0083             DTEMR(I)=DTEMP(I)*(460.0D0+TEMPO)+TEMPO
0084                EQUE(I)=-QUE
0085       75    EQUR(I)= EQUE(I)*(460.0D0+TEMPO)*THCON/DIS
            C
            C ___________________________________________________________________
            C I      CALCULATE INST. AND MEAN QUANTITIES FOR TEMP. AND HEAT T.COFF.I
            C I__________________________________________________________________I
            C
0086                HT(1)=0.0D0
0087                TGM(1)=0.0D0
0088                QM(1)=0.0D0
0089                DTWM(1)=0.0D0
0090                TWM(1)=0.0D0
0091                HM(1)=0.0D0
0092                SUMTW=0.0D0
```

```
  106 FORMAT(' NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=',I2)
      WRITE (6,109) NB
      WRITE(6,107)
      DO 15 I=1,NP
 15      WRITE(6,108) TIME(I),TEMP(I)
      WRITE (6,130) TSHFT
  109 FORMAT (' NUMBER OF B(I) COFF. TO BE FITTED = ',I2)
 130  FORMAT(' TIME OF DATA SHIFTED BY(SEC)' ,F10.6)
  108    FORMAT(' ',7X,F20.10,6X,F20.10)
  107 FORMAT('-',13X,'TIME',5X,'TEMPERATURE')
      DO 41 I=1,NB
  229 FORMAT (' COEFFICIENTS OF B(I)=', D16.8, '      I =',I2)
  41  WRITE (6,229) B(I),I
      WRITE (6,198)
      WRITE (6,199)
      WRITE(6,113)
  113 FORMAT (30X,'  TEMPERATURE AND HEAT FLUX '/,30X,'  PREDICTED BY',
     1 '   JENQ-SHING CHIOU ',///,2X,
     2'(NORTM) (REALTM) (NOR.FLUX) (REAL.FLUX) (NOR.T)   (REAL.SUF.T) (
     3TH.CO.T)')
      WRITE (6,115)
  115 FORMAT(' ','(DMLESS) (SEC.)      (DMLESS) (BTU/FTSQ SEC) (DMLESS)
     1(DEG.F.) (DEG.F.)',/)
      TEM = TEMIN
      TINC=TEM
      TRINC= TEMIN/TR1
      TREM=TRINC
      DO 90 I=1,200
         WRITE (6,114) TEM,TREM,EQUE(I),EQUR(I),DTEMP(I),DTEMR(I),TEMC
     1(I)
  114 FORMAT (F9.6,F11.7,2(F10.4),2(F12.4),F15.8)
         TEM=TEM+TINC
         TREM=TREM+TRINC
 90      CONTINUE
      WRITE(6,113)
      WRITE (6,115)
      WRITE (6,199)
      GO TO 1
C
C
C ___________________________________________________________________
C I   THIS IS THE END OF THE MAIN PROGRAM                            I
C I__________________________________________________________________I
C
  999 WRITE(6,1000)
 1000 FORMAT('-ALL DATA PROCESSED-EXECUTION OF EXECUTION')
      DEBUG SUBCHK
      END
```

```
                  SUMTG=0.0D0
                  SUMQ    =0.0D0
                  SUMDT=0.0D0
            TG(1)=(TG(1)-TEMPO)/(460.0D0+TEMPO)
            DO 80 I=1,200
                  F=I
                  SUMQ    = SUMQ     +EQUE(I)
                  CM(I) = SUMQ/F
                  SUMTG= SUMTG + TG(I)
                  TGM(I)= SUMTG/F
                  SUMDT = SUMDT + TG(I)- DTEMP(I)
                  SUMTW = SUMTW + DTEMP(I)
                  TWM(I)=SUMTW/F
                  HM(I)= SUMQ    /SUMDT
                  HT(I)=EQUE(I)/( TG(I) - DTEMP(I))
              TG(I+1)=(DTEMP(I)+DTEMP(I+1))/2.0D0+(EQUE(I)+EQUE(I+1))/(2.0D0*
            1          HT(I))
       80       CONTINUE
      C
      C ________________________________________________________________________
      C I          CALCULATE THERMOCOUPLE TEMPERATURE FROM FITTED EQ.           I
      C I_______________________________________________________________________I
      C
            TEM = TEMIN
            DO 78 I=1,200
            TEMC(I) = 0.0D0
            XX=0.5D0/DSQRT(TEM)
            CALL PERFC (NB2,BERFC,XX)
               DO 76 J=1,NB
               J2=J*2.0D0
            TCT(J)=(TEM*4.0D0)**J*BERFC(J2)*DGAMMA(J+1.0D0)
            TEMC(I) = TEMC(I) + B(J)*TCT(J)
       76      CONTINUE
            TEM = TEM + TEMIN
            TEMC(I)=TEMC(I)*(TEMPO+460.0D0)+TEMPO
       78   CONTINUE
      C
      C ________________________________________________________________________
      C I            NOW OUTPUT THE DATA                                        I
      C I_______________________________________________________________________I
      C
            WRITE (6,198)
            WRITE (6,199)
            WRITE(6,101) BORRAD
      101   FORMAT('-BORE RADIUS  (FT.) =',F10.5)
            WRITE (6,119) OUTR
      119   FORMAT  (' OUTER RADIUS (FT.)=',F10.5)
            WRITE(6,102) DIS
      102   FORMAT(' BORE TO THERMOCOUPLE  DISTANCE (FT)=',F10.6)
            WRITE(6,103) TEMPO
      103   FORMAT(' INITIAL THERMOCOUPLE TEMPERATURE (F.)=',F10.4)
            WRITE (6,301) TG(1)
      301   FORMAT (' INITIAL GAS TEMPERATURE (F.)=',F10.4)
            WRITE(6,104) ALP
      104   FORMAT (' THERMAL DIFFUSIVITY  (FTSQ/SEC) =',F15.8)
            WRITE(6,105) THCON
      105   FORMAT(' THERMAL  CONDUCTIVITY(BTU/FT.SEC.F.)=',F15.8)
            WRITE(6,106) NP
```

```
FORTRAN IV G LEVEL  21              PERFC            DATE = 76157       15/33/00            PAGE 0001

0001            SUBROUTINE PERFC(NP,BERFC,X)
          C
          C
          C ---------------------------------------------------------------
          C I             THIS SUBROUTINE CALCULATES THE REPEATED INTERGRALS  I
          C I       OF THE COMPLEMENTARY ERROR FUNCTION                       I
          C I                                                                 I
          C I       NP= NUMBER OF REPEATS INTEREALS TO BE CALCULATED          I
          C I       CERFC= REAL*8 ARRAY FOR ERROR F.                          I
          C I       X= THE INITIAL VALUE FOR ERROR FUNCTION                   I
          C I                                                                 I
          C I_________________________________________________________________I
          C
          C
0002            IMPLICIT REAL*8(A-H,O-Z)
0003            DIMENSION BERFC(52)
0004            DATA SRIPI/ 0.56418958355/
          C
          C ---------------------------------------------------------------
          C I             INITIALIZE & CALCULATE I(1)ERFC &I(2)ERFC DEPENDING I
          C I             ON NP                                               I
          C I_________________________________________________________________I
          C
0005            CALL ERRSET(208,0,-1,1,1)
0006            DO 11 I =1,50
0007            BERFC(I) =0.0D0
0008         11 CONTINUE
0009            X2=X*X
0010            BERFC1=SRIPI*DEXP(-X2)-X*DERFC(X)
0011            BERFC(1)=BERFC1
0012            IF(NP.EQ.1) RETURN
0013            BERFC2=0.25D0*((1.0D0+2.0D0*X2)*DERFC(X)-2.0D0*SRIPI*X*DEXP(-X2))
0014            BERFC(2)=BERFC2
0015            IF(NP.EQ.2)  RETURN
          C
          C ---------------------------------------------------------------
          C I             NOW GO INTO DO LOOP & CALCULATE I(NP)ERFC           I
          C I_________________________________________________________________I
          C
0016            DO 10 I=3,NP
0017            BERFC(I)=0.5D0/I*(BERFC1-2.0D0*X*BERFC2)
0018               BERFC1=BERFC2
0019         10    BERFC2=BERFC(I)
0020            DEBUG SUBCHK
0021            RETURN
0022            END
```

```
0001            SUBROUTINE  LTSQER (M,JJ,PHI,B,EPS,TIME,NT,TRAN,QTQ,TEMPO)
          C
          C ______________________________________________________________________
          C I   THIS PROGRAM CALCULATES THE LEAST EQUARES COEFFICIENTS FOR         I
          C I   MODIFIED POLYNOMIAL WHICH IS POLYNOMIAL TIMES THE INTEGRATED       I
          C I   ERROR FUCTION                                                      I
          C I                                                                      I
          C I   M IS THE NUMBER OF DATA PAIRS TO BE USED                           I
          C I   JJ IS THE NUMBER OF DEGREE TO BE FITTED                            I
          C I______________________________________________________________________I
          C
          C
0002            REAL*8 PHI(26,26),FX(26),TRAN(26,26),QTQ(26,26),X(26),DET, FPX,
               1B(26),TIME(26)
0003            CALL ERRSET(208,0,-1,1,1)
0004            LDIM=50
0005            N1=JJ+1
0006            DO 1001 I=1,M
0007            X(I)=TIME(I)
0008            FX(I)=PHI(I,N1)
0009       1001 CONTINUE
0010            ERROR =0.000
          C
          C ______________________________________________________________________
          C I   FIND THE TRANSPOSE OF THE PHI     MATRIX FOR LSTSQ F TTING AND     I
          C I   PERFORM THE MULTIPLICATION    AND GET JJ BY JJ MATRIX              I
          C I______________________________________________________________________I
          C
0011            CALL TRANS (PHI, TRAN,M,JJ,NT)
0012            CALL MULT (PHI,TRAN,JJ,M,QTQ,NT)
0013            DO 4 J=1,JJ
0014            QTQ(J,N1)=0.000
0015            DO 4 K=1,M
0016          4 QTQ(J,N1)=QTQ(J,N1) + TRAN(J,K)*FX(K)
0017            WRITE (6,5) (I,QTQ(I,N1),I=1,JJ)
0018          5 FORMAT (' I=',I2,'    QTQ(I,N1)=',D16.7)
          C
          C ______________________________________________________________________
          C I   FIND THE SOLUTION TO THE NORMAL EQUATIONS                          I
          C I______________________________________________________________________I
          C
0019            DET = SIMUL (JJ,QTQ,B,EPS,1,20)
0020            WRITE (6,100)
0021        100 FORMAT ('  LEAST SCUARE COEFFICIENTS ')
0022            WRITE (6,6) (I,B(I)   ,I=1,JJ)
0023          6 FORMAT (' I= ',I2,  '  B(I)= ', D16.7)
0024            WRITE (6,9)
0025          9 FORMAT (//' LEAST SQ OUT PUT ',//' KTH TIME',T15,'TEMP. GIVEN ',
               1T30,'FITTED', T42, 'ERROR',T55,'PER CENTAGE')
0026            DO 15 K=1,M
0027            FPX=TEMPO
0028            DO 8 I=1,JJ
0029          8 FPX=FPX + QTQ(I,N1)* PHI(K,I)
0030            FX(K)= FX(K) + TEMPO
0031            ERR = PHI(K,N1) - FPX + TEMPO
0032            IF (PHI(K,N1).NE. 0.)  PER=ERR/(PHI(K,N1) + TEMPO)*100.0
0033            IF( ABS(ERR).LE.ERROR) GO TO 15
0034            ERROR = ABS(ERR)
```

```
 15   WRITE (6,17) K,X(K),FX(K), FPX,ERR,PER
 17   FORMAT (1I3,    3(F10.5,5X),F10.6,3X,F6.2)
      WRITE (6,110)   ERROR
110   FORMAT ('  MAXIMUN ERROR ='   ,D16.7)
      DEBUG SUBCHK
      RETURN
      END

      SUBROUTINE TRANS (PH1,TRAN,M,N,N1)
C
C ____________________________________________________________________
C I      THIS SUBROUTINE GIVE THE TRANSPOSE OF THE MATRIX
C I___________________________________________________________________I
      REAL*8 PH1(26,26),TRAN(26,26)
      CALL ERRSET(208,0,-1,1,1)
      DO 10 J=1,N
      DO 10 I=1,M
 10   TRAN(J,I)=PH1(I,J)
      RETURN
      END


      SUBROUTINE MULT (PH1,TRAN,N,M,QTQ,N1)
C
C ____________________________________________________________________
C I      THIS SUBROUTINE GIVES MULTIPLICATION OF TWO MATRIES          I
C I___________________________________________________________________I
      REAL*8 PH1(26,26),TRAN(26,26),QTQ(26,26)
      CALL ERRSET(208,0,-1,1,1)
      DO 1 I=1,N
      DO 1 J=1,N
      QTQ(I,J)=0.000
      DO 1 K=1,M
  1   QTQ(I,J)=QTQ(I,J)+TRAN(I,K)*PH1(K,J)
      RETURN
      END
```

```
0001            FUNCTION SIMUL(N,A,X,EPS,INDIC,NRC)
          C
          C ----------------------------------------------------------------
          C I    .....GAUSS JORDAN TECHNIQUE MAXIMUM  PIVOT                  I
          C I    ....PAGE 290 'APPLIED NUMERICAL METHODS' CARNAHAN           I
          C I________________________________________________________________I
          C
0002            IMPLICIT REAL*8(A-H,O-Z)
0003            DIMENSION IROW(60),JCOL(60),JORD(60),Y(60),A(26,26  ),X(26)
0004            CALL ERRSET(208,0,-1,1,1)
0005            MAX=N
0006            IF(INDIC.GE.0) MAX=N+1
          C     ....IS N LARGER THAN 50
0007            IF(N.LE.50) GO TO 5
0008            WRITE (6,200)
0009            SIMUL=0.0D0
0010            RETURN
          C     BEGIN ELIMINATION PROCEDURE
0011      5     DETER=1.0D0
0012            DO 18 K=1,N
0013            KM1=K-1
          C    .....SEARCH FO R THE PIVOT ELEMENT
0014            PIVOT=0.0D0
0015            DO 11 I=1,N
0016            DO 11 J=1,N
          C    ....SCAN IROW AND JCOL ARRAYS FOR INVALID PIVOT SUBSCRIPTS
0017            IF(K.EQ.1) GO TO 9
0018            DO 8 ISCAN=1,KM1
0019            DO 8 JSCAN=1,KM1
0020            IF(I.EQ.IROW(ISCAN)) GO TO 11
0021            IF(J.EQ.JCOL(JSCAN)) GO TO 11
0022      8     CONTINUE
0023      9     IF(DABS(A(I,J)).LE.DABS(PIVOT)) GO TO 11
0024            PIVOT=A(I,J)
0025            IROW(K)=I
0026            JCOL(K)=J
0027      11     CONTINUE
          C     ....INSURE THAT SELECTED PIVOT IS LARGER THAN EPS
0028            IF(DABS(PIVOT).GT.EPS) GO TO 13
0029            SIMUL=0.0D0
0030            RETURN
          C     UPDATE THE DETERMINANT VALUE...
0031      13    IROWK=IROW(K)
0032            JCOLK=JCOL(K)
0033            DETER=DETER*PIVOT
          C     ....NORMALIZE PIVOT ROW ELEMENTS
0034            DO 14 J=1,MAX
0035      14     A(IROWK,J)=A(IROWK,J)/PIVOT
          C    ...CARRY OUT ELIMATION AND DEVELOPE INVERSE
0036            A(IROWK,JCOLK)=1./PIVOT
0037            DO 18 I=1,N
0038            AIJCK=A(I,JCOLK)
0039            IF(I.EQ.IROWK) GO TO 18
0040            A(I,JCOLK)=-AIJCK/PIVOT
0041            DO 17 J=1,MAX
0042      17    IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
0043      18       CONTINUE
          C   ....ORDER SOLUTION VALUES AND CREATE JORD ARRAY
```

```
           DO 20 I=1,N
           IROWI=IROW(I)
           JCOLI=JCOL(I)
           JORD(IROWI)=JCOLI
      20   IF(INDIC.GE.0) X(JCOLI)=A(IROWI,MAX)
      C    ADJUST SIGN OF DETERMINANT
           INTCH=0
           NM1=N-1
           DO 22 I=1,NM1
           IP1=I+1
           DO 22 J=IP1,N
           IF(JORD(J).GE.JORD(I))  GO TO 22
           JTEMP=JORD(J)
           JORD(J)=JORD(I)
           JORD(I)=JTEMP
           INTCH=INTCH+1
      22   CONTINUE
           IF(INTCH/2*2.NE.INTCH) DETER=-DETER
      C    ....IF INDIC IS POSITIVE RETURN WITH RESULTX
           IF(INDIC.LE.0) GO TO 26
           SIMUL=DETER
           RETURN
      C    .....IF INDIC IS NEGATIVE OR ZERO, UNSCRAMBLE YHE INVERSE
      C    ...FIRST BY ROWS
      26   DO 28 J=1,N
           DO 27 I=1,N
           IROWI=IROW(I)
           JCOLI=JCOL(I)
      27   Y(JCOLI)=A(IROWI,J)
           DO 28  I=1,N
      28   A(I,J)=Y(I)
      C    ...THEN BY COLUMNS
           DO 30 I=1,N
           DO 29 J=1,N
           IROWJ=IROW(J)
           JCOLJ=JCOL(J)
      29   Y(IROWJ)=A(I,JCOLJ)
           DO 30 J=1,N
      30   A(I,J)=Y(J)
      C    .....RETURN FOR INDIC NEGATIVE OR ZERO
           SIMUL=DETER
           RETURN
      C    ....FORMAT OFR OUTPUT STATEMENT
      200  FORMAT(10HON TOO BIG)
           DEBUG SUBCHK
           END
```

APPENDIX III M60 DATA

```
                    M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH
                     BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM
NUMBER OF B(I) COFF. TO BE FITTED = 10

                      TIME                    TEMPERATURE

                   0.0100000000             110.6000000000
                   0.0200000000             124.5000000000
                   0.0300000000             123.1000000000
                   0.0400000000             119.6000000000
                   0.0500000000             116.9000000000
                   0.0600000000             114.1000000000
                   0.0700000000             112.5000000000
                   0.0800000000             110.3000000000
                   0.0900000000             107.6000000000
                   0.1000000000             106.8000000000
TIME OF DATA SHIFTED BY(SEC) -0.002000
BORE RADIUS  (FT.) =    0.01250
OUTER RADIUS (FT.)=   0.04417
BORE TO THERMOCOUPLE  DISTANCE (FT)=  0.001830
INITIAL THERMOCOUPLE TEMPERATURE (F.)=    78.8000
INITIAL GAS TEMPERATURE (F.)=     4.4937
THERMAL DIFFUSIVITY  (FTSQ/SEC) =        0.00010307
THERMAL  CONDUCTIVITY(BTU/FT.SEC.F.)=       0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=10
NUMBER OF B(I) COFF. TO BE FITTED = 10

                    M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREECH
                     BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM
NUMBER OF B(I) COFF. TO BE FITTED = 10

                      TIME                    TEMPERATURE

                   0.0100000000             126.2000000000
                   0.0200000000             131.6000000000
                   0.0300000000             124.2000000000
                   0.0400000000             119.6000000000
                   0.0500000000             116.8000000000
                   0.0600000000             113.3000000000
                   0.0700000000             109.8000000000
                   0.0800000000             107.1000000000
                   0.0900000000             105.7000000000
                   0.1000000000             104.2000000000
TIME OF DATA SHIFTED BY(SEC) -0.002000
BORE RADIUS  (FT.) =    0.01250
OUTER RADIUS (FT.)=   0.03562
BORE TO THERMOCOUPLE  DISTANCE (FT)=  0.001670
INITIAL THERMOCOUPLE TEMPERATURE (F.)=    78.8000
INITIAL GAS TEMPERATURE (F.)=     4.4937
THERMAL DIFFUSIVITY  (FTSQ/SEC) =        0.00010307
THERMAL  CONDUCTIVITY(BTU/FT.SEC.F.)=       0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=10
NUMBER OF B(I) COFF. TO BE FITTED = 10
```

M60 GUN THERMOCOUPLE 4 9.0 INCHES FROM BREECH
BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.0100000000	143.2000000000
0.0200000000	157.3000000000
0.0300000000	151.0000000000
0.0400000000	144.4000000000
0.0500000000	137.5000000000
0.0600000000	132.8000000000
0.0700000000	129.0000000000
0.0800000000	125.9000000000
0.0900000000	122.8000000000
0.1000000000	119.6000000000

TIME OF DATA SHIFTED BY(SEC) -0.002000
BORE RADIUS (FT.) = 0.01250
OUTER RADIUS (FT.)= 0.05000
BORE TO THERMOCOUPLE DISTANCE (FT)= 0.001830
INITIAL THERMOCOUPLE TEMPERATURE (F.)= 78.8000
INITIAL GAS TEMPERATURE (F.)= 4.4937
THERMAL DIFFUSIVITY (FTSQ/SEC) = 0.00010307
THERMAL CONDUCTIVITY(BTU/FT.SEC.F.)= 0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=10
NUMBER OF B(I) COFF. TO BE FITTED = 10

APPENDIX IV. THE CASE OF OSCILATORY SURFACE TEMPERATURE

Consider a slab with a sufficient thickness, ℓ, such that when a surface is subjected to a periodic surface temperature variation with a frequency w the other surface is held at the initial temperature T_o. If properties are assumed constant the governing equation for the problem can be written as

$$\frac{\partial \nu}{\partial t} = \alpha \frac{\partial^2 \nu}{\partial x^2} \tag{1}$$

with initial and boundary conditions as

$$\nu(x, o) = 0 \tag{2}$$

$$\nu(\ell, t) = \sin wt \tag{3}$$

$$\nu(o, t) = 0 \tag{4}$$

where $\nu = (T - T_o)/(T_{max} - T_o)$ and α is the thermal diffusivity.

The solution of the problem according to Carslaw and Jaeger [4] can be written as

$$\nu = 2\alpha\pi \sum_{n=1}^{\infty} (-1)(-1)^n n \left[(\alpha n^2\pi^2 \sin wt - w\ell^2 \cos wt) + w\ell^2 e^{-\alpha n^2\pi^2 t/\ell^2}\right] \cdot \sin\left(\frac{n\pi x}{\ell}\right) / [\alpha^2 n^4 \pi^4 + w^2 \ell^2] \tag{5}$$

Part III PREDICTION OF TRANSIENT SURFACE HEAT FLUX AND TEMPERATURE ON A HOLLOW CYLINDER

I. INTRODUCTION

In the study of transient heat transfer many efforts have been made on the so-called "inverse problem" [1,2] where a surface heat flux and temperature is to be predicted by the measured data at some location interior to a body.

In the previous works [1-6] the solution is represented in either an integral form after some manipulation of the contour integral from the inverse transform, or in a series form after the expansion of the solution for small and large times. Using Laplace transformation Chen and Thomsen [6] introduced a polynomial in terms of the error function to represent the response of thermocouple measurement and the inversion is accomplished for any transient surface heat flux at the inner surface of a cylindrical tube. However, their inversion solution is valid only for a short duration due to the asymptotic expansion of the modified Bessel function in the inverse Laplace transform. In this study an exact solution obtained from the inverse Laplace transform by the convolution method is given for the case of hollow cylinder. The solution is valid for both constant and variable heat flux and for both short and long time duration.

II. ANALYSIS

Consider a long hollow cylinder with sufficient wall thickness such that the outer surface temperature has a negligible response when the inner surface is exposed to a thermal pulse of a transient process. This condition considerably simplifies the theoretical analysis as the outer boundary may be assumed to be infinite, and only one interior probe of the cylinder

is required in the experimental measurement. The material of the cylinder is considered to be homogeneous and isotropic with constant thermal diffusivity, α. Let R_i and R_o be, respectively, the inner and outer surface radii. R_1 the radius of the probe location and t the dimensionless time. If the temperature of the cylinder is initially uniform at T_o, the mathematical problem governing the temperature T, may be written as

$$\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} \qquad 1 < r < r_o = \infty \tag{1}$$

$$\theta\ (r, o) = 0 \tag{2}$$

$$\theta\ (\infty, t) = 0 \tag{3}$$

$$\theta\ (r_1, t) = f(t) \qquad 1 < r_1 < \infty \tag{4}$$

where $\theta = T - T_o$, $r = R/R_i$, $t = \alpha\tau/R_i^2$, and f(t) is the interior temperature response of the thermocouple measured at $r = r_1$ at the dimensionless time t. The problem is to predict the surface temperature θ (1, t) and heat flux per unit area

$$q = -\ (K/R_1)(\partial\theta/\partial r)\Big|_{r=1} \tag{5}$$

where K is the thermal conductivity.

The problem can be solved by Laplace transformation. Let the transformation be

$$\bar{\theta}(r, s) = \int_o^\infty \theta e^{-ts} dt \tag{6}$$

when θ satisfies the Dirichlet's condition the temperature function θ is recovered by inversion of the Laplace transformation as

$$\theta(r,\ t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \bar{\theta} e^{st} ds \qquad (7)$$

where c is a suitable positive value. Equation (1) and (2) under transformation (6) becomes

$$\frac{d^2\bar{\theta}}{dr^2} + \frac{1}{r}\frac{d\bar{\theta}}{dr} = s\bar{\theta}$$

which has a solution of the form

$$\bar{\theta} = AI_o(pr) + BK_o(pr) \qquad (8)$$

where I_o and K_o are modified Bessel functions of the first and second kind with $p = (s)^{1/2}$. With the boundary conditions (3) and (4), (8) becomes

$$\bar{\theta} = \bar{f}(s)\ [K_o(pr)/K_o(pr_1)] \qquad (9)$$

where $\bar{f}(s)$ is the Laplace transform of the boundary condition (4).

The temperature response measured at $r = r_1$ can be expresses by a polynomial or numerous other suitable functions. In the present analysis, for reasons to be explained later, f(t) will be represented as

$$f(t) = \sum_{n=1}^{N} b_n \int_o^t F_1(\tau)\ Fn(t - \tau)d\tau \qquad (10)$$

If we choose $F_1(t) = \frac{1}{2\tau} e^{-\frac{r_1^2}{4\tau}}$ and $Fn(t - \tau)$ being any arbitrary function

depending on n, for example $(t - \tau)^n$ etc. then the Laplace transform of Eq. (10) gives

$$\bar{f}(s) = \sum_{n=1}^{N} b_n K_o(pr_1)\bar{F}n(s) \tag{11}$$

where $\bar{F}n(s)$ is the Laplace transform of $Fn(t)$. Substituting Eq. (11) into Eq. (19) we have

$$\bar{\theta}(s, r) = \sum_{n=1}^{N} b_n K_o(pr)\bar{F}n(s) \tag{12}$$

It is noted that the $K_o(pr_1)$ in Eq. (9) has been cancelled by this substitution which explains the choice of $F(t) = \frac{1}{4K} e^{-\frac{r_1^2}{4K}}$ in Eq. (10). The inversion of Eq. (12) gives

$$\theta(t, r) = \sum_{n=1}^{N} b_n \int_o^t \frac{1}{2\tau} e^{-\frac{r^2}{4\tau}} Fn(t - \tau)d\tau \tag{13}$$

At surface $r = 1$

$$\theta(t, 1) = \sum_{n=1}^{N} b_n \int_o^t \frac{1}{2\tau} e^{-\frac{1}{4\tau}} Fn(t - \tau)d\tau \tag{14}$$

The temperature gradient and hence heat flux at surface is

$$\left.\frac{\partial\theta}{\partial r}\right|_{r=1} = -\sum_{n=1}^{N} b_n \int_o^t \frac{1}{4\tau^2} e^{-\frac{1}{4\tau}} Fn(t - \tau)d\tau \tag{15}$$

some examples of $Fn(t - \tau)$ frunction and the representation of the thermocouple response are:

Case 1 If $\bar{F}n(s) = \dfrac{1}{s^2+n^2}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_o^t \frac{1}{2\tau}\, e^{-\frac{r_1^2}{4\tau}}\, \frac{1}{n} \sin n(t - \tau)d\tau \qquad (16)$$

Case 2 If $\bar{F}n(s) = \dfrac{s}{s^2+n^2}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_o^t \frac{1}{2\tau}\, e^{-\frac{r_1^2}{4\tau}} \cos n(t - \tau)d\tau \qquad (17)$$

Case 3 If $\bar{F}n(s) = \dfrac{1}{(s + a)^n}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_o^t \frac{e^{-\frac{r_1^2}{4\tau}} (t - \tau)^{n-1} e^{-a(t-\tau)}}{2\tau(n - 1)} d\tau \qquad (18)$$

Case 4 If $\bar{F}n(s) = s^{-(n+1/2)}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_o^t \frac{e^{-\frac{r_1^2}{4\tau}}\, 2^n (t - \tau)^{n-1/2}}{2\tau\ 1.3.5\ldots(2n - 1)\sqrt{\pi}} d\tau \qquad (19)$$

Case 5 If $\bar{F}n(s) = \dfrac{1}{s^n}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_o^t e^{-\frac{r_1^2}{4\tau}}\, \frac{(t - \tau)^{n-1}}{2\tau(n - 1)!} d\tau \qquad (20)$$

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